ISSN 2229-5518

Attribute Delocalization Observation

Aakash Kumar

Abstract
In this paper, the concept of "Attribute Delocalization Observation" has been proposed which provides a better method to understand logical mathematics specially probability.

The concept is developed by considering an observation that "An event and its complements don't exist individually as a sample point rather they are attributes of the same sample point".

According to this concept "An event and its complements are considered as the attributes of sample points. Fractional count ratio (it is defined as, if in an urn, there are m white balls and n black balls, then fractional count ratio of white attribute = m/(m + n) and fractional count ratio of black attribute = n/(m+ n))gives the portion in which those attributes lie inside the sample points. Probability of an event is equal to the fractional count ratio of the event".

By using this concept, probability can be understood with greater ease which would reduce complexity. In this paper some of the typical questions have been solved by using the existing concept as well as using the proposed concept entitled "Attribute Delocalization Observation". A comparative analysis has been made to show how easily these problems can be solved with greater insight by using the proposed concept.

The paper has also made an attempt to include some challenging questions which can be easily solved by applying this concept only.
Key words: attribute, delocalization, event, fractional count ratio, sample point, urn, ball.

1. INTRODUCTION

Urn problem is the intellectual way of representing real objects by the colour of balls in an urn(s). The probabilistic approach of real object is correlated with movement of different coloured balls from one urn to another with care whether the movement is done with replacement or without replacement. As per Laplace's great theory [1]:

The Probability for an event is the ratio of the nos. of cases favourable to it, to the nos. of all cases possible when nothing leads us to expect that any one of these cases should occur more than any other, which renders them, for us, equally Possible.

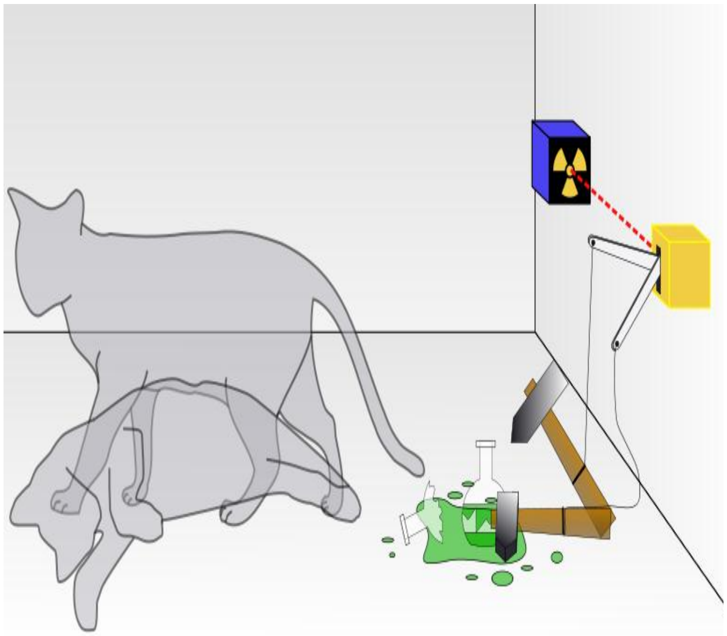
The understanding of the concept of probability can be better and easier if the proposed definition of "**Attribute delocalization observation**" can serve as a complementary definition to the existing definition of Laplace theory.

The proposed definition of **Attribute delocalization observation** is:
"An event and its complements are treated as attributes of sample points. All attributes are delocalized and reside within each sample point in their fractional count ratio.

If individual attribute within each sample point is p, then probability of event corresponding to that individual attribute will be p".

By Schrödinger's Cat

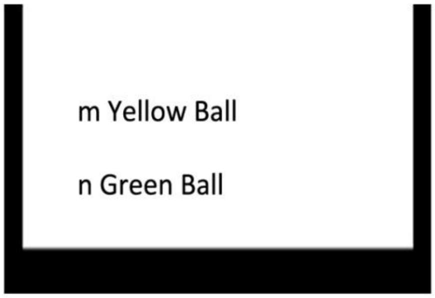
PROOF OF THIS OBSERVATION IS INSPIRED

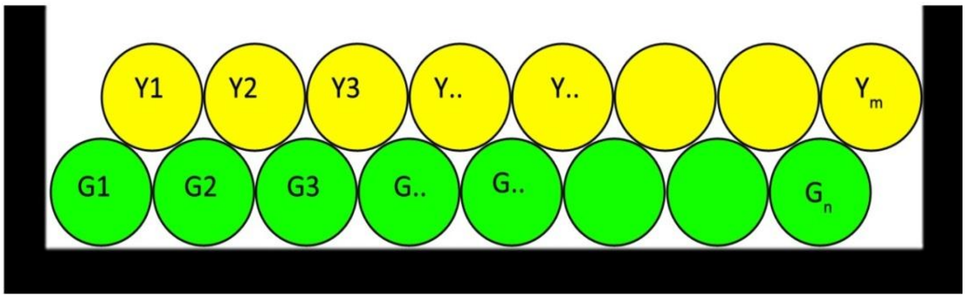


A cat, along with a flask containing a poison and a radioactive source, is placed in a sealed box. If an internal monitor detects radioactivity, the flask is shattered, releasing the poison that kills the cat. The Copenhagen interpretation of quantum mechanics implies that after a while, the cat is *simultaneously* alive *and* dead. Yet, when we look in the box, we see the cat *alive or* dead, not both alive *and* dead [2].

2. EXPLANATION OF THIS OBSERVATION BY EXAMPLE:

Let us consider a Bag containing m yellow balls and n green balls. Then it can be assumed that the bag has m yellow attributes and n green attributes.



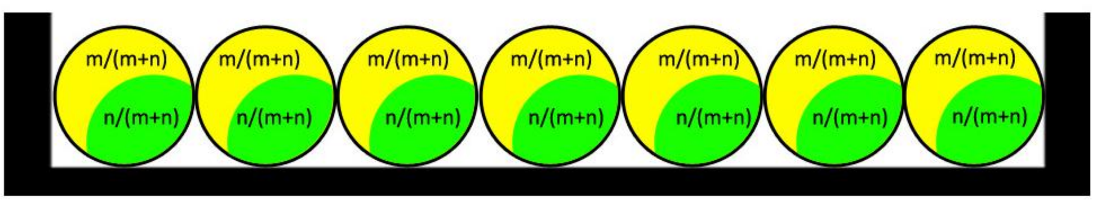


ATTRIBUTE DELOCALIZATION EXPLANATION-1:

Overall ball's attributes = (m + n) units.

It can be assumed that yellow attributes and green attributes are delocalised within all the (m + n) balls in their fractional count ratio.

That means each ball contains $\frac{m}{m+n}$ yellow attributes and $\frac{n}{m+n}$ green attributes.



Initial

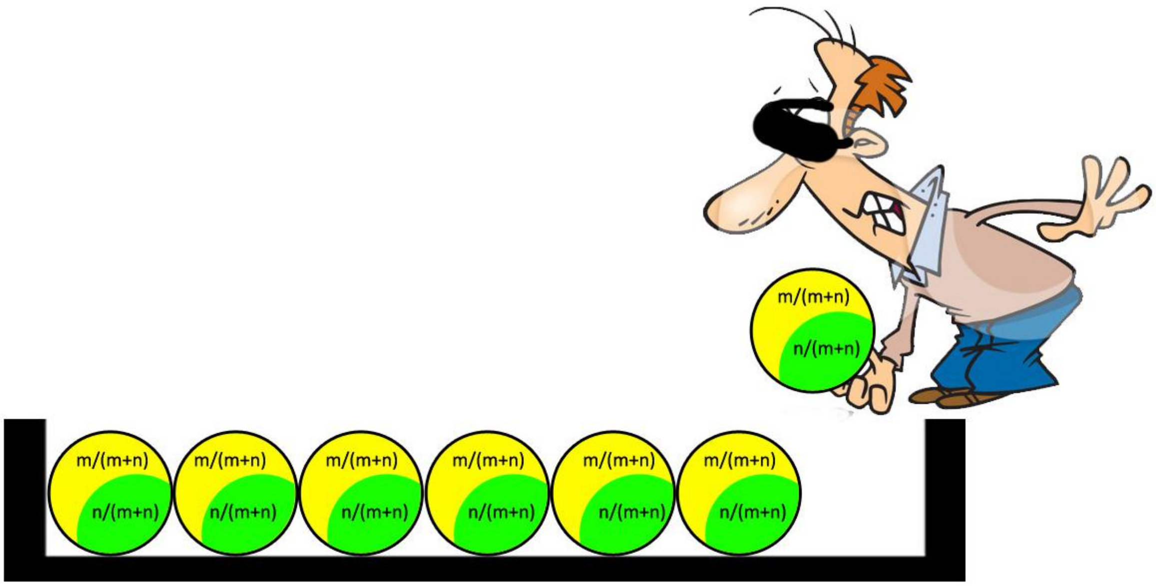
Fractional count ratio of yellow attribute $=\frac{m}{m+n}$

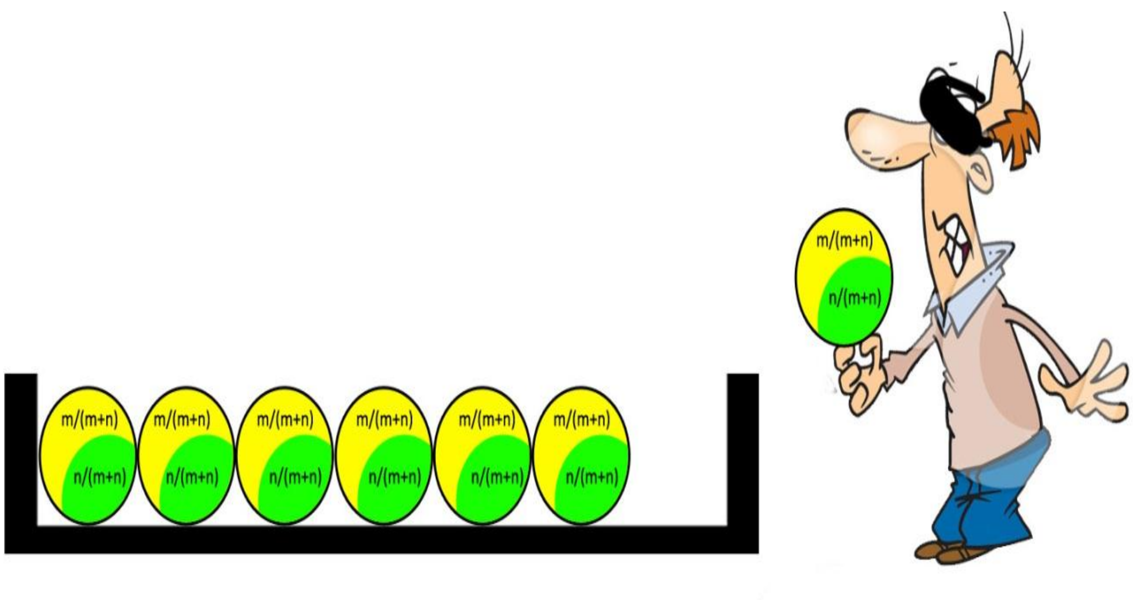
Fractional count ratio of green attribute $=\frac{n}{m+n}$

Hence probability of drawing a Ball having yellow colour (attributes) $=\frac{m}{m+n}$, and green colour (attributes) $=\frac{n}{m+n}$

It is known that probability is guess before the knowledge of result.

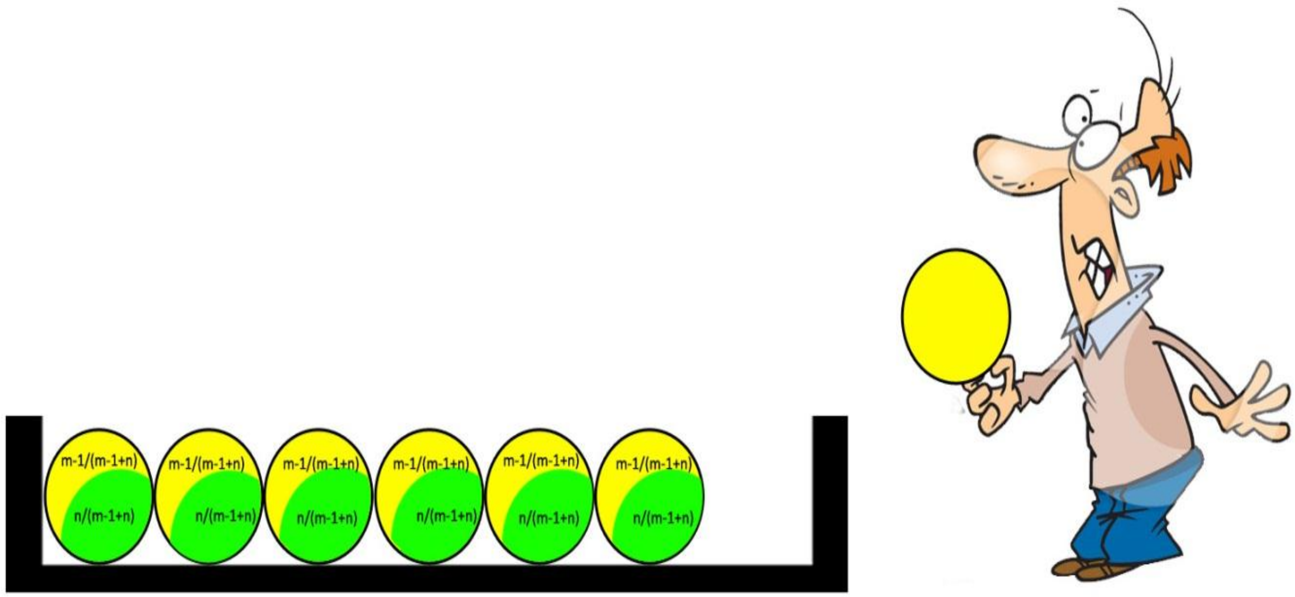
It can be understood from the following example: If a man picks a ball from the bag with his eyes closed. Then it can be thought that he hasn't picked up a yellow ball or a green ball, rather it can be thought that he picks a ball in which all its attributes reside in their fractional count ratio.





As long as he doesn't see the ball, the ball is *simultaneously* yellow *and* green. When he opens his eyes, he observes that the ball is *either* yellow *or* green, not both yellow *and* green.

ISSN 2229-5518



Case 1

Case1: when the drawn ball is found yellow then for the remaining (m+n-1) balls person will be blind.

That is among remaining (m+n-1) balls

Total no. of yellow balls = (m-1)

Total no. of green balls= n.

From attribute delocalization observation, all yellow attributes

(m-1) and green attributes (n) will be delocalised within all the remaining (m + n - 1) balls.

Fractional count ratio of yellow attributes $=\frac{m-1}{(m-1)+n}$

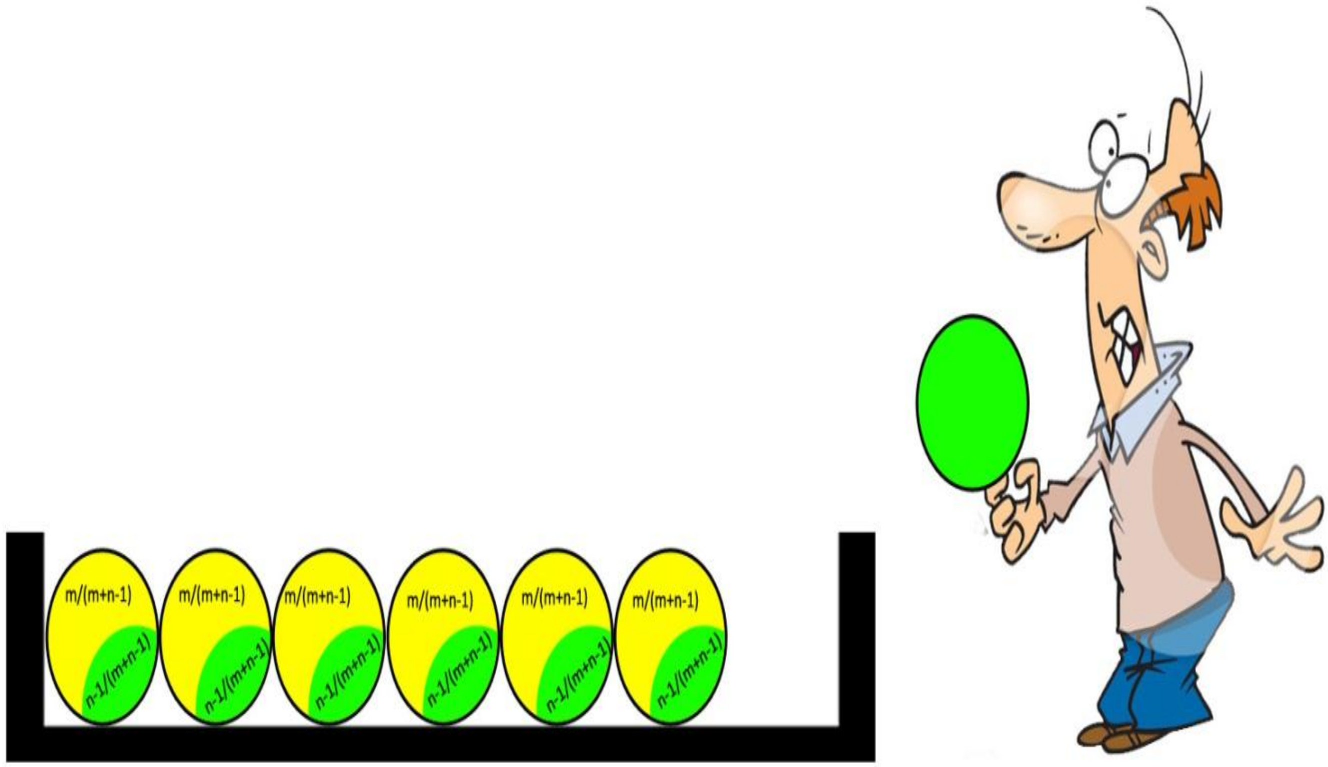
Fractional count ratio of green attributes$=\frac{n}{(m-1)+n}$

Now each ball contains $\frac{m-1}{(m-1)+n}$ yellow ball and $\frac{n}{(m-1)+n}$ green ball.

Balls may be fractional for the calculation of probability.

Hence, probability of drawing a ball having yellow colour (attributes) = $\frac{\boldsymbol{m-1}}{\boldsymbol{(m-1)+n}}$ and green colour (attributes) $=\frac{\boldsymbol{n}}{\boldsymbol{(m-1)+n}}$.

ISSN 2229-5518

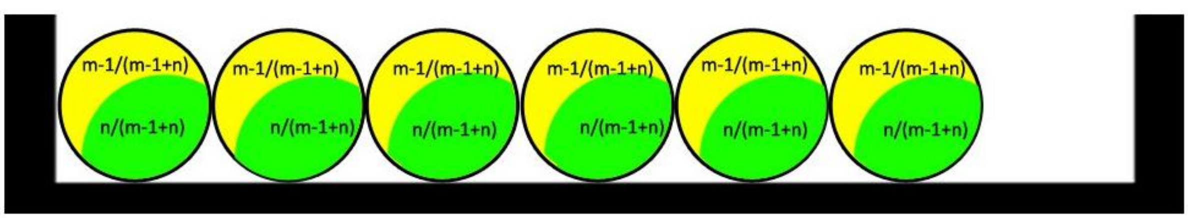


Case 2

Case 2: When the drawn ball is found green, then basing on the above argument fractional count ratio of yellow is $\frac{m}{m+(n-1)}$ and green is $\frac{n-1}{m+(n-1)}$ and hence corresponding probabilities

.**Consider the situation after one ball is drawn and let that ball be yellow.**

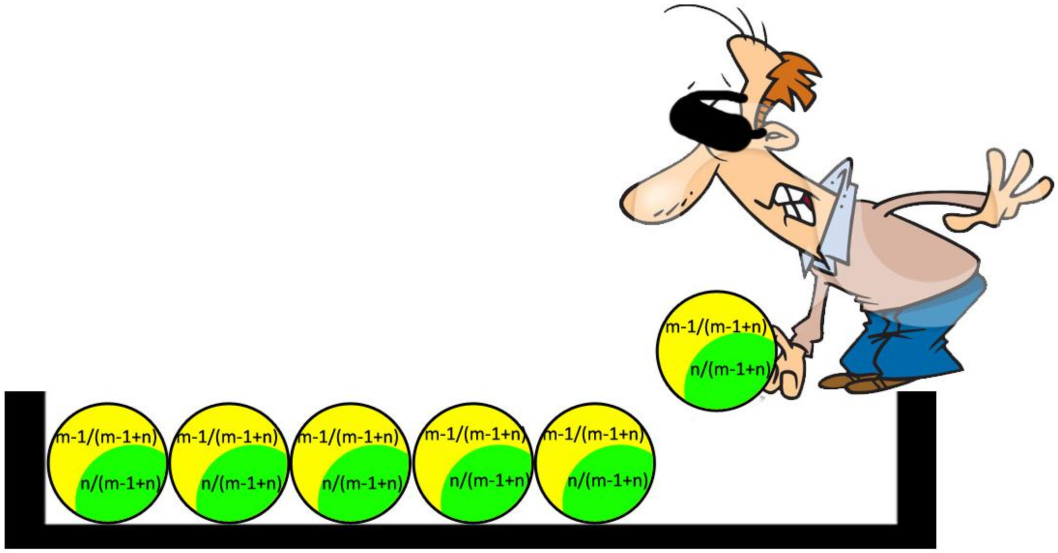
Then the situation would be like

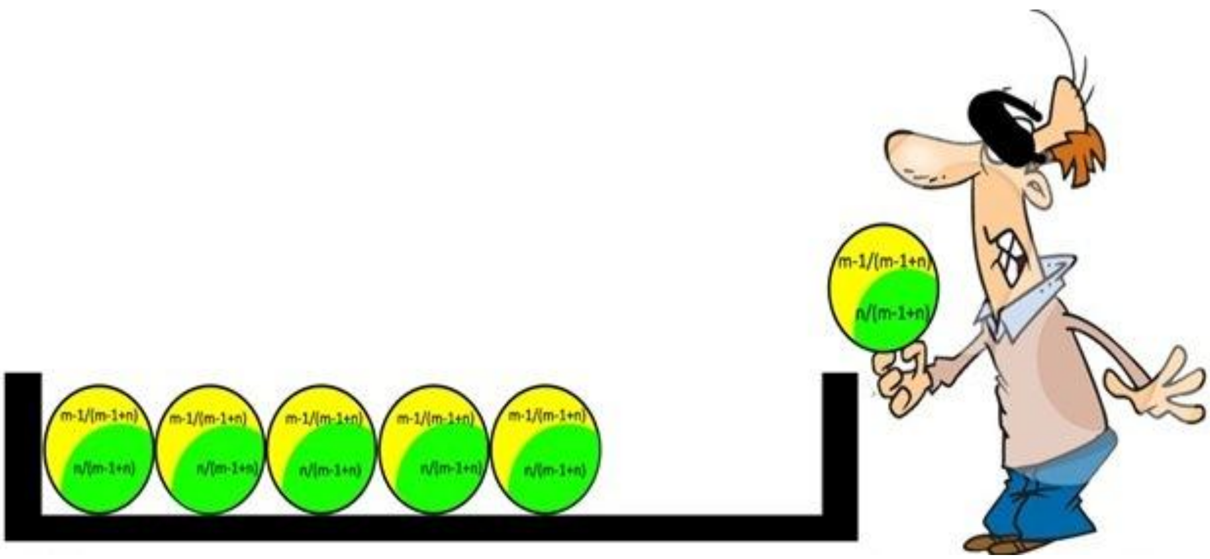


Now when a man picks a ball from the remaining (m+n-1) balls,

Then the situation is

ISSN 2229-5518

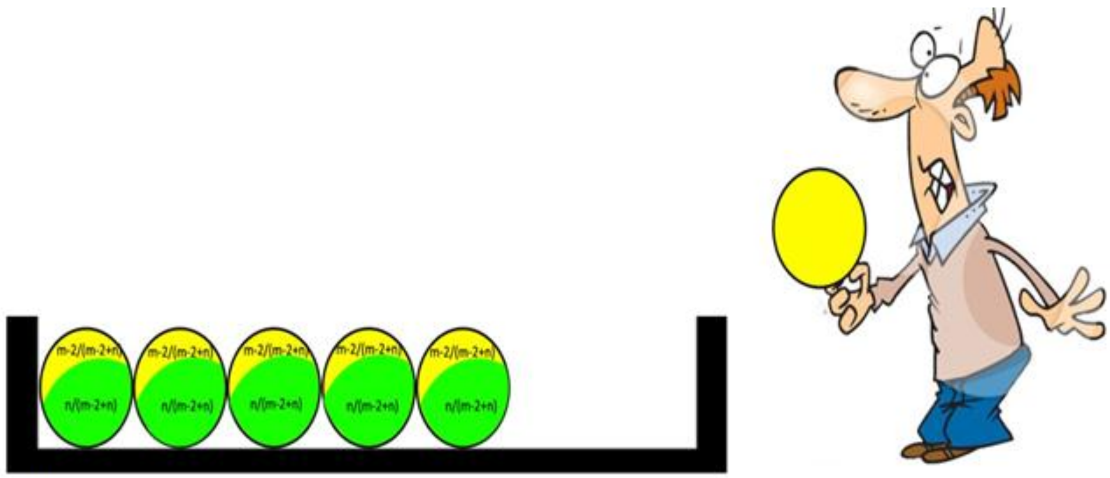




As long as he doesn't see the ball, the ball is *simultaneously* yellow *and* green.

When he opens his eyes, he observes that the ball is *either* yellow *or* green, not both yellow *and* green.

ISSN 2229-5518



Case 1

Case1: when the drawn ball is yellow then for remaining (m+n-2) balls person will be blind.

That is among remaining (m+n-2) balls

Total no. of yellow balls = (m-2)

Total no. of green balls = n.

From attribute delocalization observation it can be understood that

All yellow attributes (m-2) and green attributes (n) will be delocalised within all the remaining (m+n-2) balls.

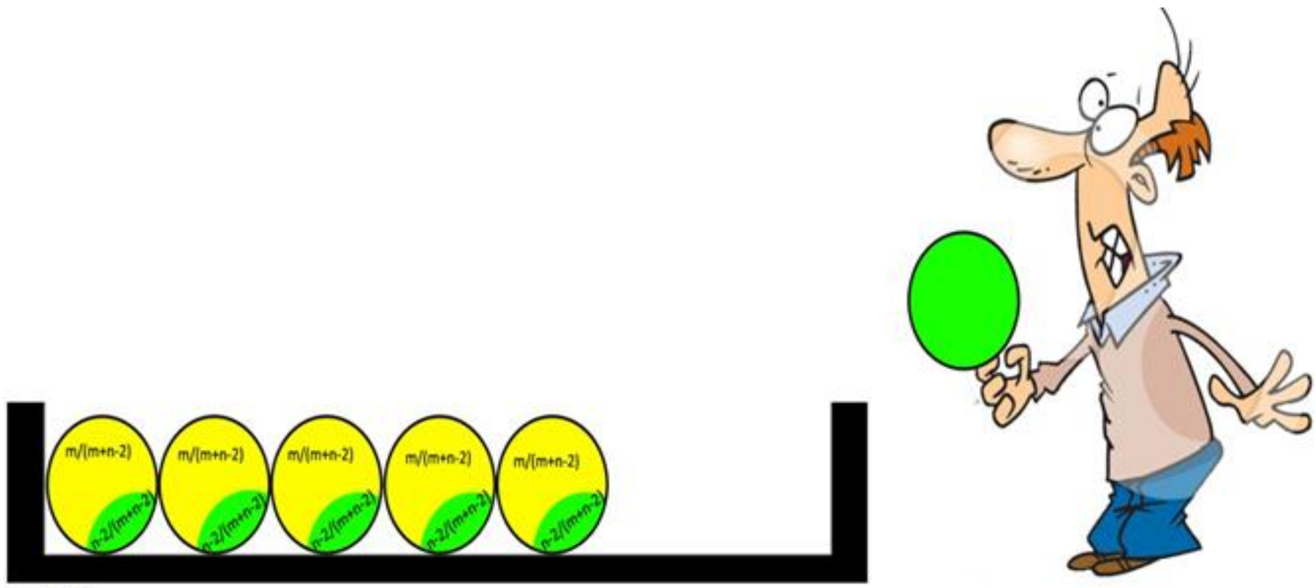
Fractional count ratio of yellow attributes $=\frac{m-2}{(m-2)+n}$

Fractional count ratio of green attributes $=\frac{n}{(m-2)+n}$

Among remaining (m+n-2) balls, each ball contains $\frac{m-2}{(m-2)+n}$ yellow attributes and $\frac{n}{(m-2)+n}$ green attributes.

It can be noted that after two yellow balls are drawn, the fractional count ratio of yellow attributes decreases whereas that of green attributes increases. **Hence, probability of drawing a ball having yellow colour (attributes) = $\frac{m-2}{(m-2)+n}$ and green colour (attributes) $=\frac{n}{(m-2)+n}$.**

ISSN 2229-5518



Case 2

Case2: when the drawn ball is found green then for remaining (m+n-2) balls person will be blind.

That means among remaining (m+n-2) balls

Total no. of yellow balls= (m-1)

Total no. of green balls = (n-1).

From attribute delocalization observation, it is understood that

All yellow attributes (m-1) and green attributes (n-1) will be delocalised within all the remaining (m+n-2) balls.

Fractional count ratio of yellow attributes $=\frac{m-1}{(m-1)+(n-1)}$ and

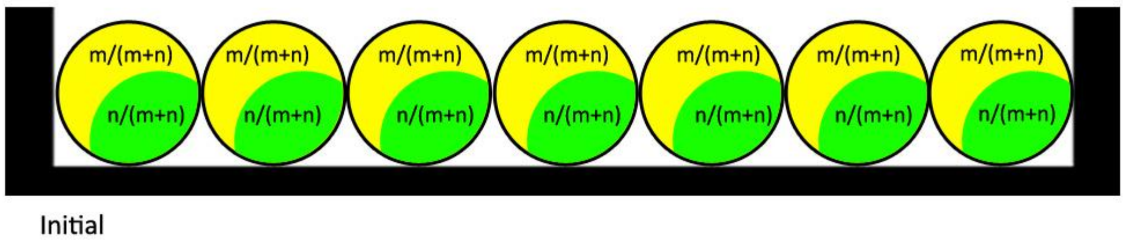
The fractional count ratio of green attributes $=\frac{n-1}{(m-1)+(n-1)}$.

Now each ball contains $\frac{m-1}{(m-1)+(n-1)}$ yellow attributes and $\frac{n-1}{(m-1)+(n-1)}$ green attributes.

Hence, probability of drawing a Ball having yellow colour (attributes) $=\frac{m-1}{(m-1)+(n-1)}$ and green colour (attributes) $=\frac{n-1}{(m-1)+(n-1)}$

ANOTHER VIEW OF ATTRIBUTE DELOCALIZATION (2nd view):

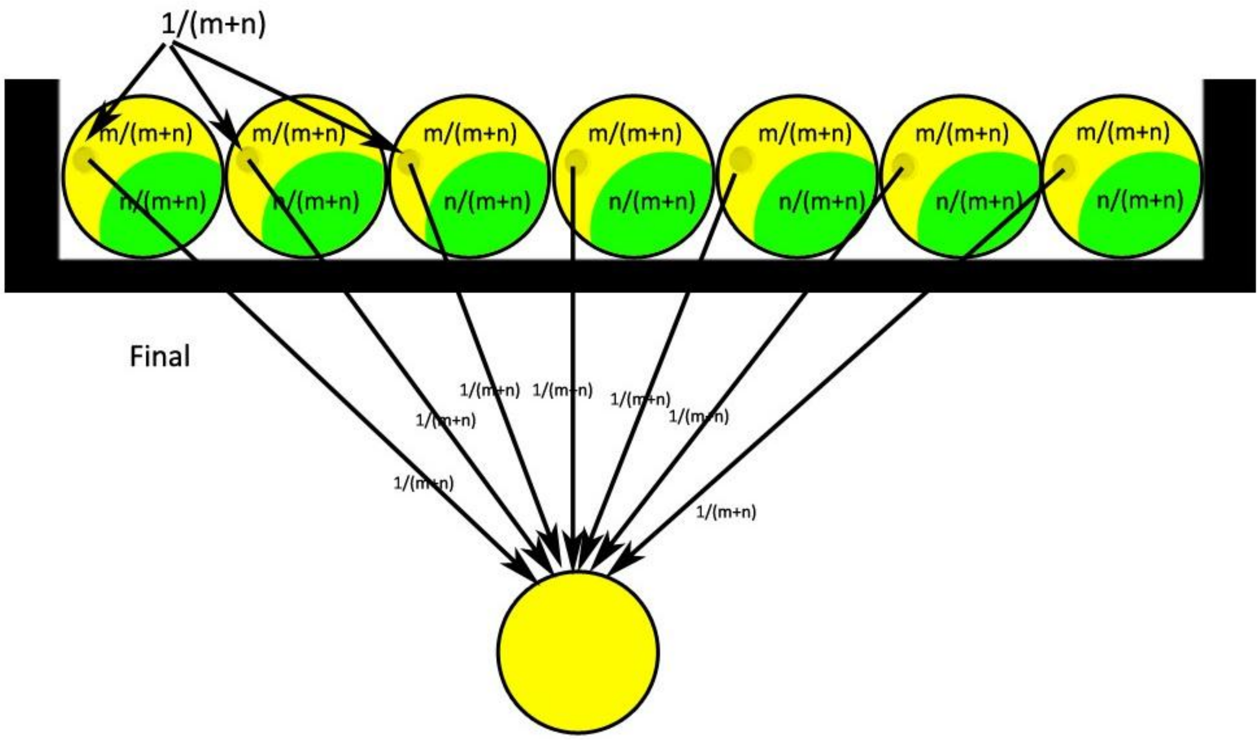
In each object concentration of yellow attributes and green attributes will be in same ratio.



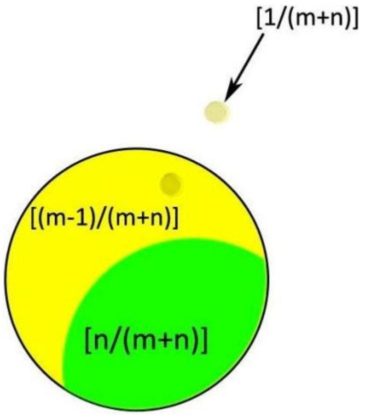
Drawing a yellow ball can be assumed as 1 unit of yellow ball donated by (m + n) balls.

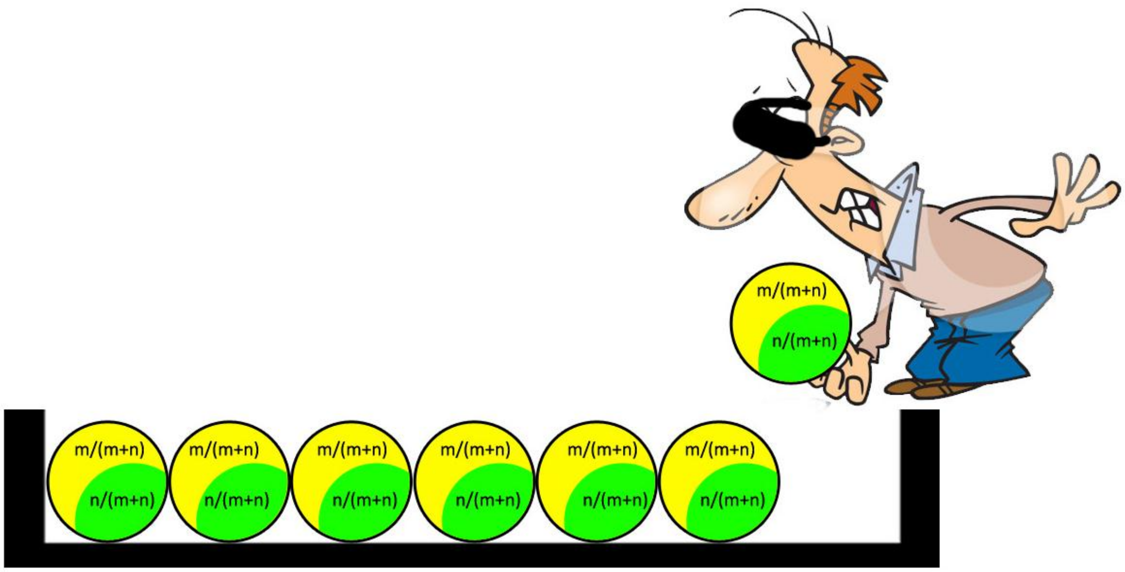
(m + n) objects donate one unit of yellow attributes.

So, 1 object donates $\frac{1}{m+n}$ unit of yellow attributes.

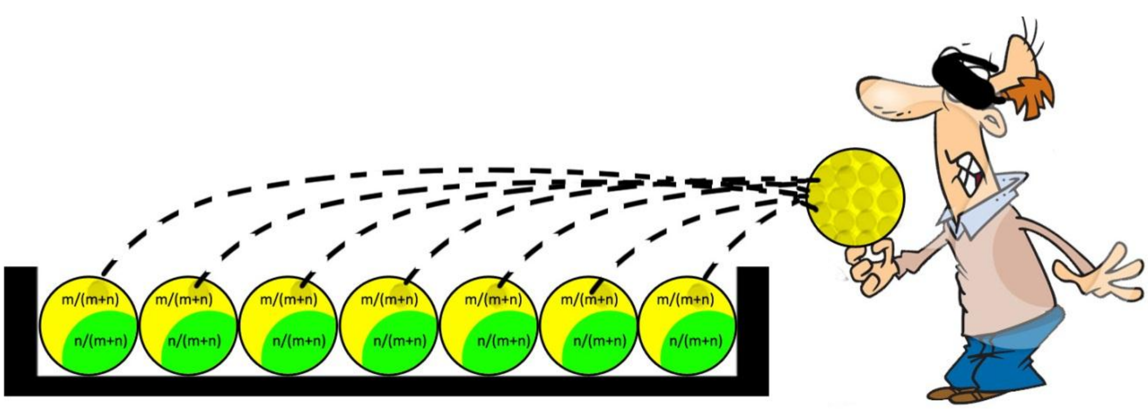


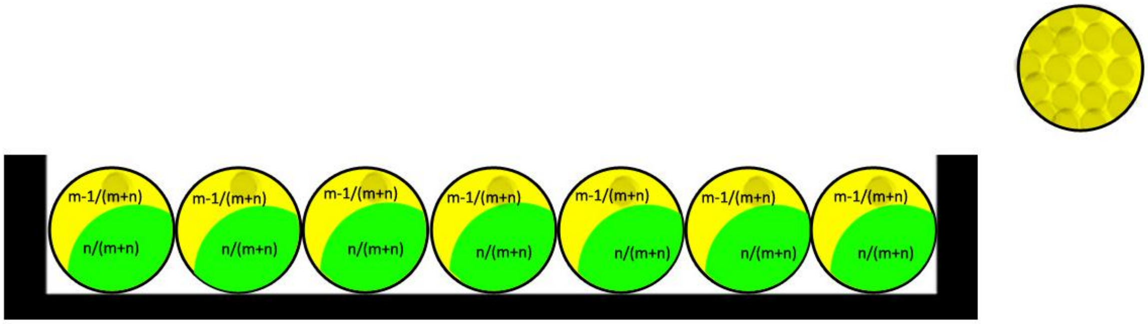
After donation, each ball has $\frac{m+n-1}{m+n}$ portion of the ball because some portion (here $\frac{1}{m+n}$) of it is removed from every ball. Now each ball has $\frac{m-1}{m+n}$ yellow ball and $\frac{n}{m+n}$ green ball.





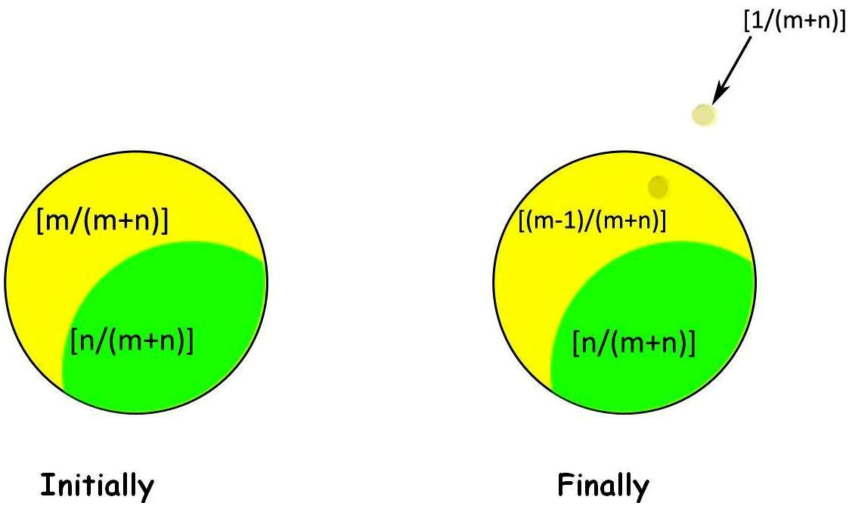
It can be noted here that the drawn yellow ball is made up of yellow portion ($\frac{1}{m+n}$) donated by each ball.



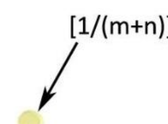


Here we **logically** assume that we still have (m + n) balls in the bag even though one ball has been drawn.

But now each ball doesn't remain the same complete ball as a portion $\frac{1}{m+n}$ is removed.

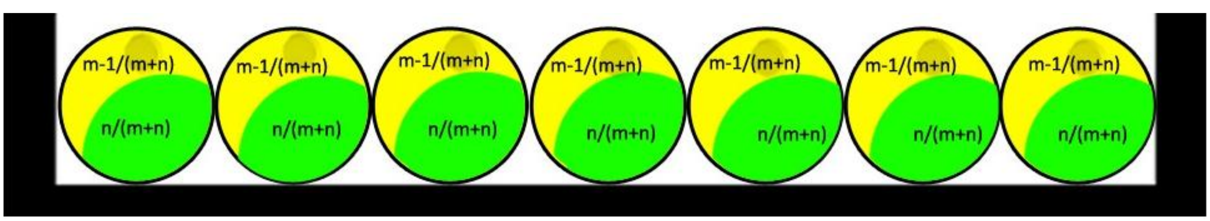


Each ball having internal configuration before and after the removal of one yellow ball

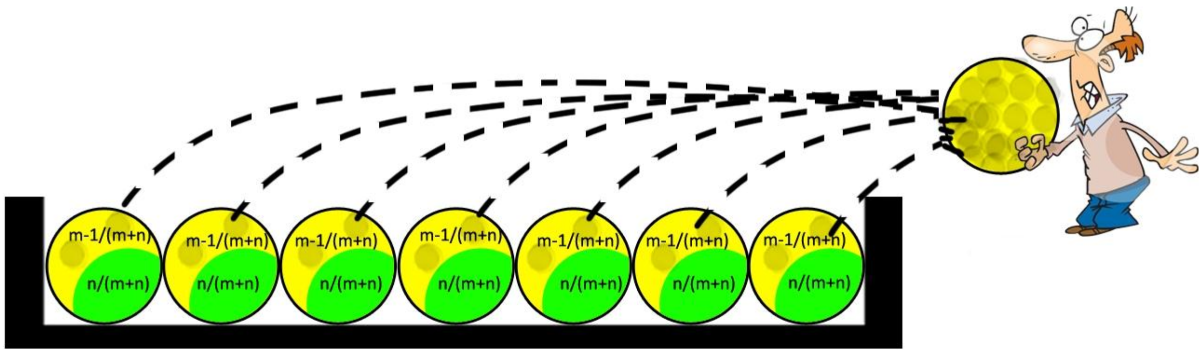
OBSERVATION: After the removal one of the balls (for example one yellow ball), there is one trench 

of that ball colour among all (m + n) balls.

➔ **Consider the situation when one yellow ball is already drawn. A person now draws another ball.**



Initially one yellow ball is already drawn.



Still contains (m+n) balls

Here total nos. of balls inside bag =m + n.

But each ball is not a complete ball as $\frac{1}{m+n}$ Portion of it has been already removed. Now each ball is represented as (1 - $\frac{1}{m+n}$) .

As each ball contains $\frac{m+n-1}{m+n}$ complete balls.

So $(m + n)$ ball contains $\frac{m+n-1}{m+n}$ *(m $+n$) complete balls.

Each ball contains ($\frac{m}{m+n}$- $\frac{1}{m+n}$) yellow attributes.

So $(m + n)$ ball contains $\frac{m-1}{m+n}$ *(m $+n$) yellow attributes.

Each ball contains ($\frac{n}{m+n}$) green attributes.

So $(m + n)$ ball contains $\frac{n}{m+n}$ *(m+n) green attributes.

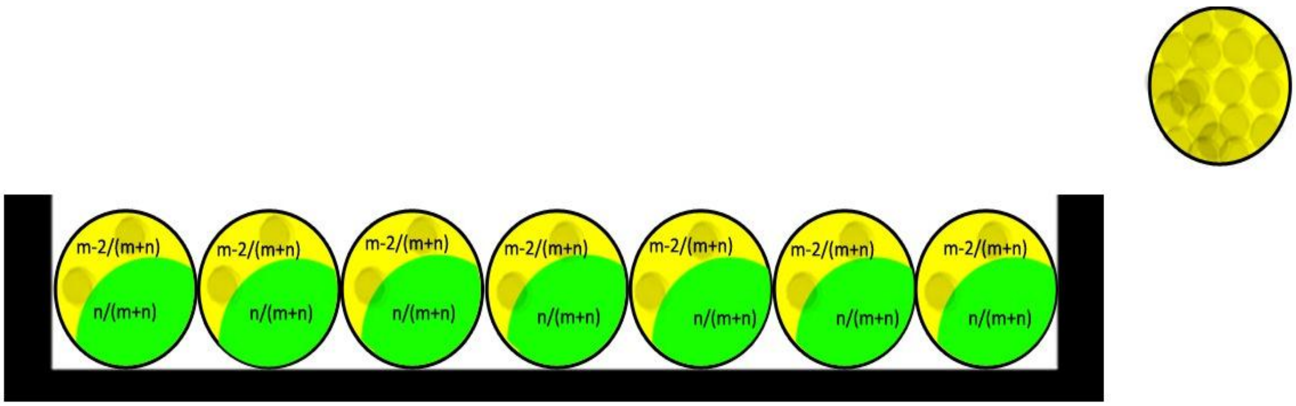
From attribute delocalization observation,

$\frac{m-1}{\boldsymbol{m+n}}$ *$(\boldsymbol{m} + \boldsymbol{n})$ yellow attribute and $\frac{n}{\boldsymbol{m+n}}$ *$(\boldsymbol{m} + \boldsymbol{n})$ green attribute is delocalized within $\frac{m+n-1}{\boldsymbol{m+n}}$ *$(\boldsymbol{m} + \boldsymbol{n})$ complete balls.

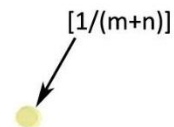
So p(Y)= $\dfrac{\frac{m-1}{\boldsymbol{m+n}} *(\boldsymbol{m+n})}{\frac{m+n-1}{\boldsymbol{m+n}} *(\boldsymbol{m+n})} = \dfrac{m-1}{m+n-1}$.

and p(G)= $\dfrac{\frac{n}{\boldsymbol{m+n}} *(\boldsymbol{m+n})}{\frac{m+n-1}{\boldsymbol{m+n}} *(\boldsymbol{m+n})} = \dfrac{n}{m+n-1}$.

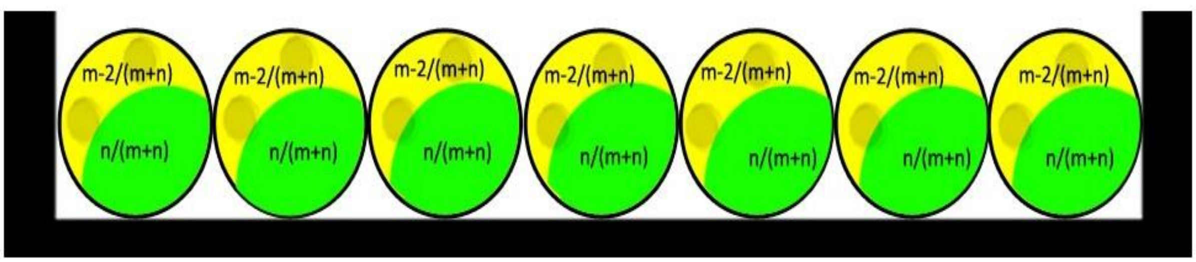
Note: p(Y) and p(G) will remain the same if we draw a ball from among **k** balls (0<**k**<=(m + n)).



Still contains (m+n) balls

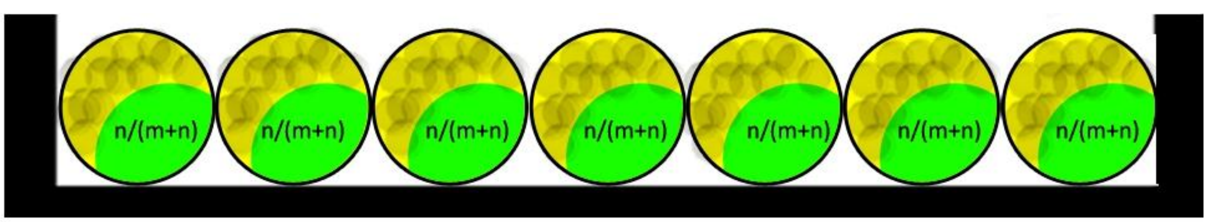
Situation after two yellow balls are drawn .Here two yellow trenchs 

are observed in all the m+n balls.



Still contains (m+n) balls

- ➔ **Consider the extreme situation when all the yellow balls have been drawn.**
  Now if one ball is drawn after the removal all the yellow balls.



As each ball now contains $\frac{n}{m+n}$ complete balls.

So $(m+n)$ balls contain $\frac{n}{m+n}$ *(m $+n$) complete balls.

Each ball contains 0 yellow attribute.

So $(m+n)$ balls contain 0 yellow attribute.

Each ball contains ($\frac{n}{m+n}$) green attributes.

ISSN 2229-5518

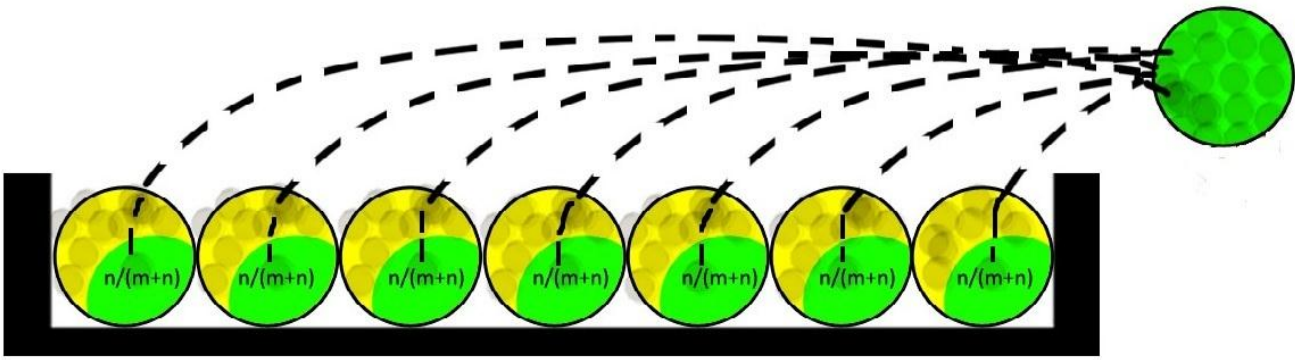
So $(m+n)$ balls contain $\frac{n}{m+n}*(m+n)$ green attributes.

From attribute delocalization observation,

0 yellow attribute and $\frac{n}{m+n}*(m+n)$ green attributes is delocalized within all $\frac{n}{m+n}*(m+n)$ complete balls.

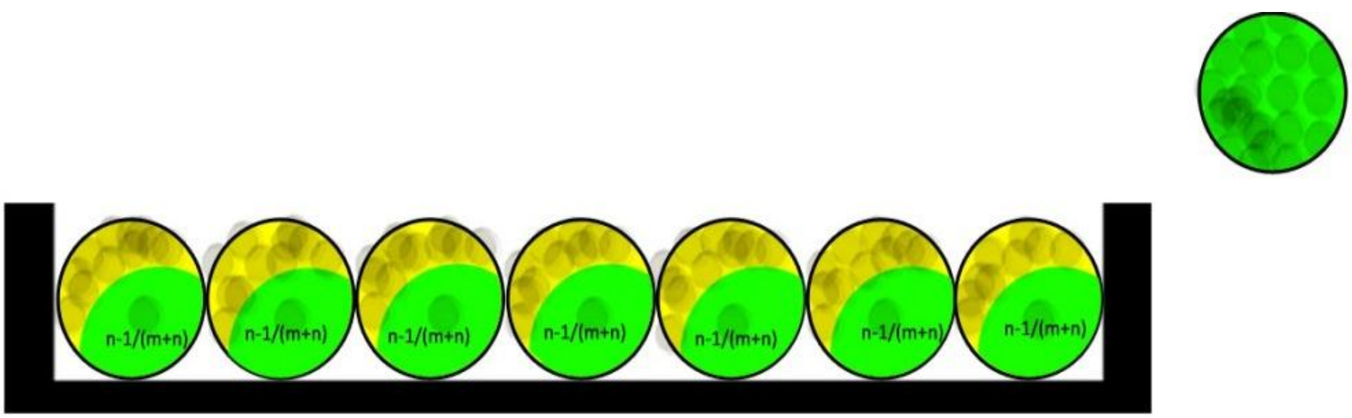
So $p(Y) = \dfrac{0}{\frac{n}{m+n}*(m+n)}$.

$P(G) = \dfrac{\frac{n}{m+n}*(m+n)}{\frac{n}{m+n}*(m+n)} = 1.$

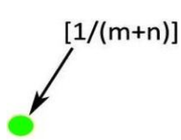


Still contains (m+n) balls

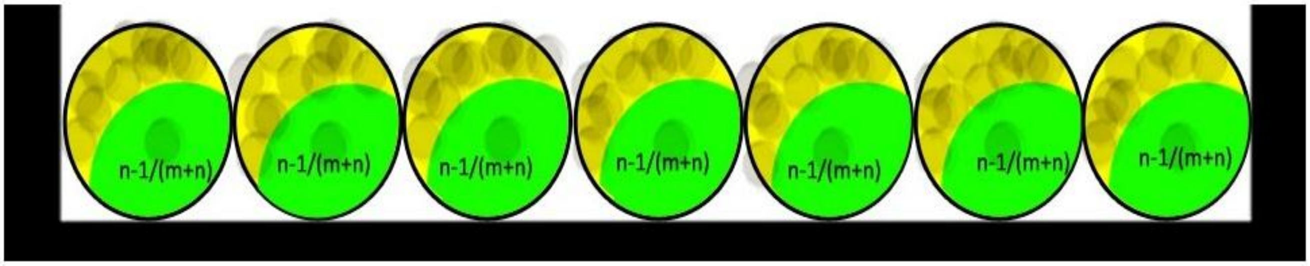
Situation after the drawn of a green ball:



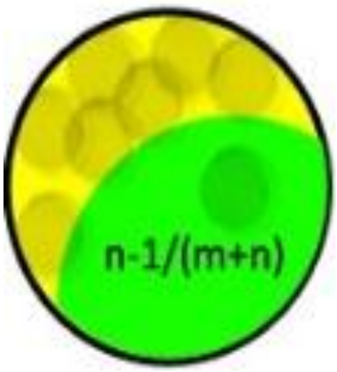
Still contains (m+n) balls

As one green ball is drawn so, one green trench 

is observed in the green portion of each ball of the bag.



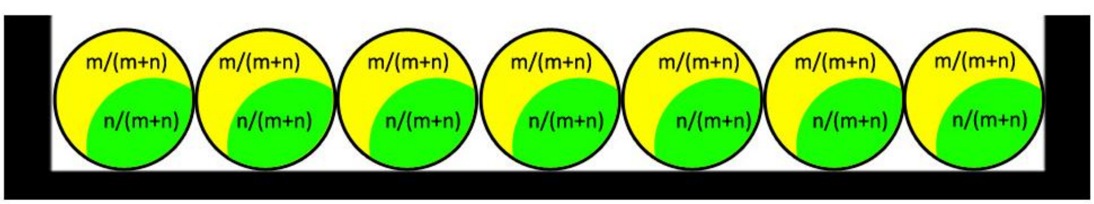
Still contains (m+n) balls



The figure above shows the internal structure of ball after the drawn of all yellow balls and a green ball.

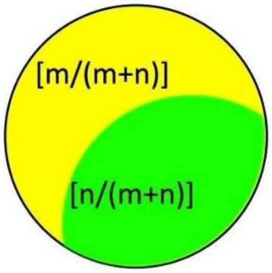
Bernoulli urn scenario sampling without replacement

In an urn there are m yellow and n green balls. What is Probability of drawing yellow at any draw if it does not know the result of any other draw?



Initial

As the result of any other draw is not known then from attribute delocalization internal configuration of balls remains the same at any draw.

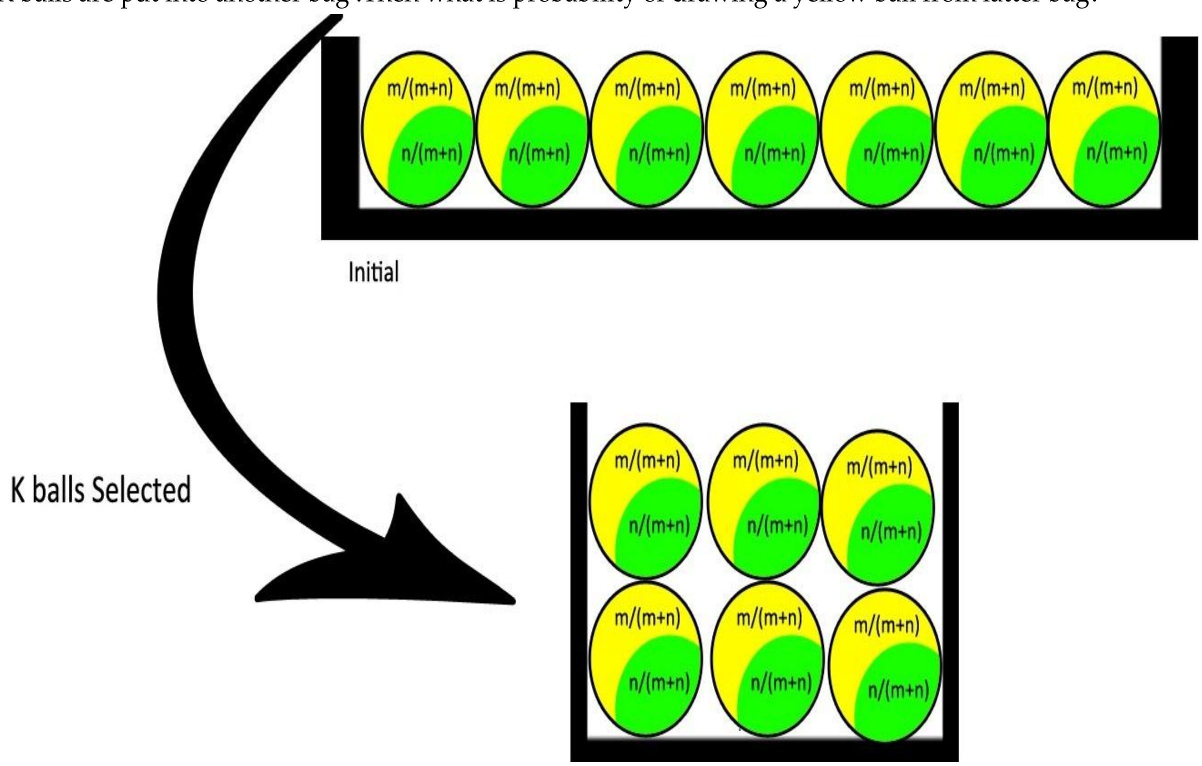


Hence, probability of drawing yellow ball at any draw if it does not know the result of any other draw = $\frac{m}{m+n}$.

Note: **if one ball is drawn in r^th attempt then also p(Y) =$\frac{m}{m+n}$ and p (G) = $\frac{n}{m+n}$ if and only if for the all Previous (r-1)^th result is not known .**

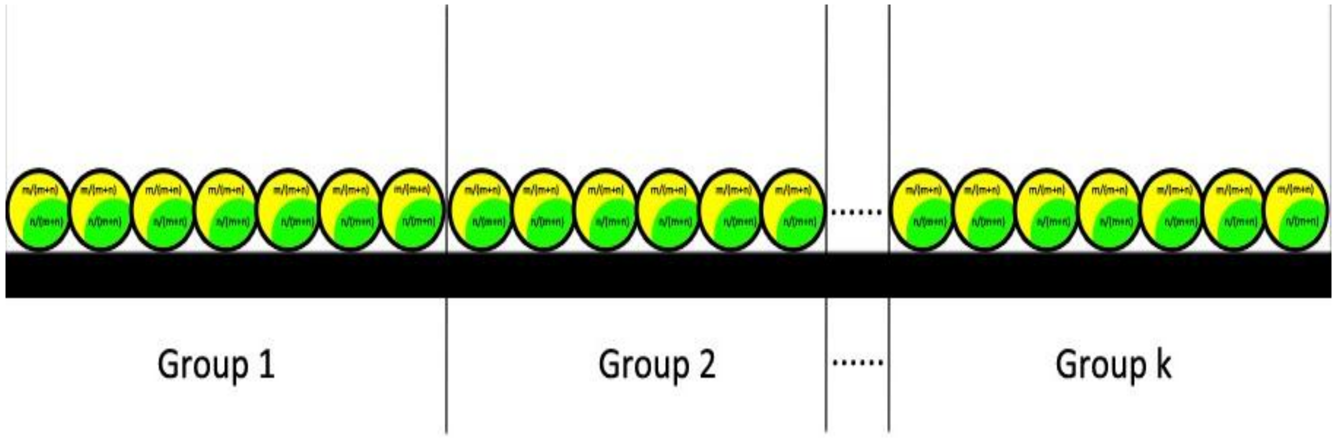
2.1 PARTITIONING HAS NO EFFECT ON PROBABILITY AS LONG AS PARTITIONING IS DONE BLINDLY.

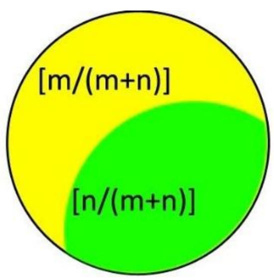
Let a bag contain m yellow balls and n green balls.
K balls are put into another bag .Then what is probability of drawing a yellow ball from latter bag?



It is similar to doing partition of bag among k groups.

ISSN 2229-5518





Internal structure of each ball.

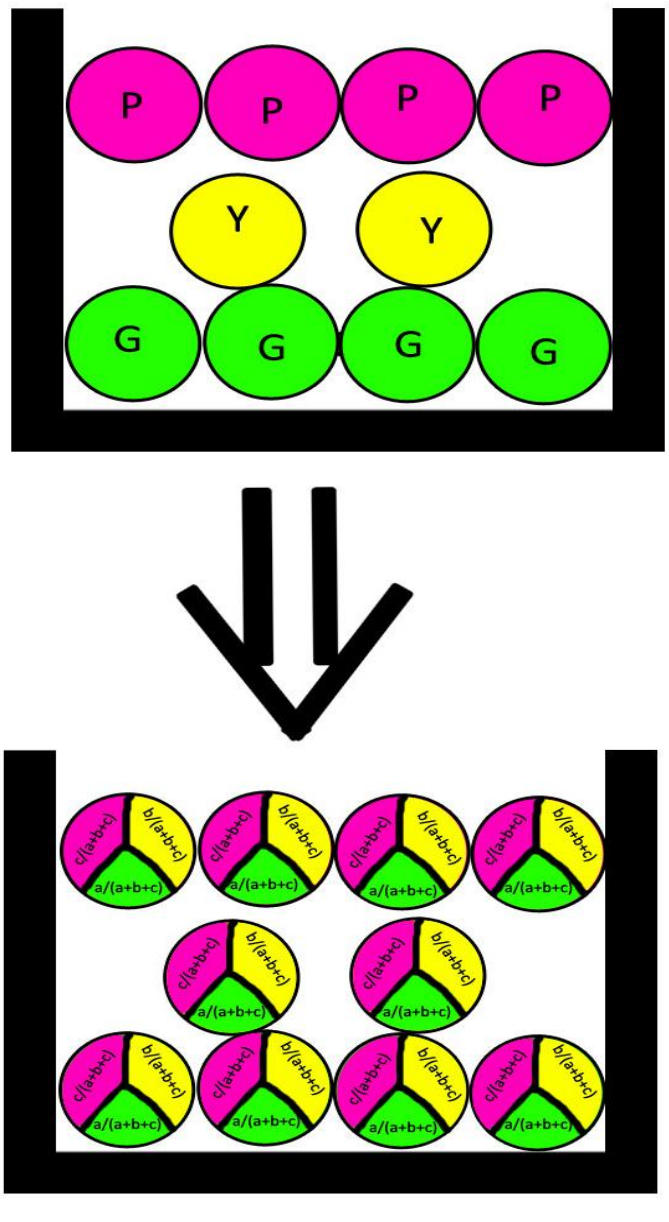
Obviously all attributes (here yellow and green) are delocalized within all the (m + n) balls.

And each ball has same internal configuration so probability of drawing a yellow ball and green ball will be same among the entire k groups.

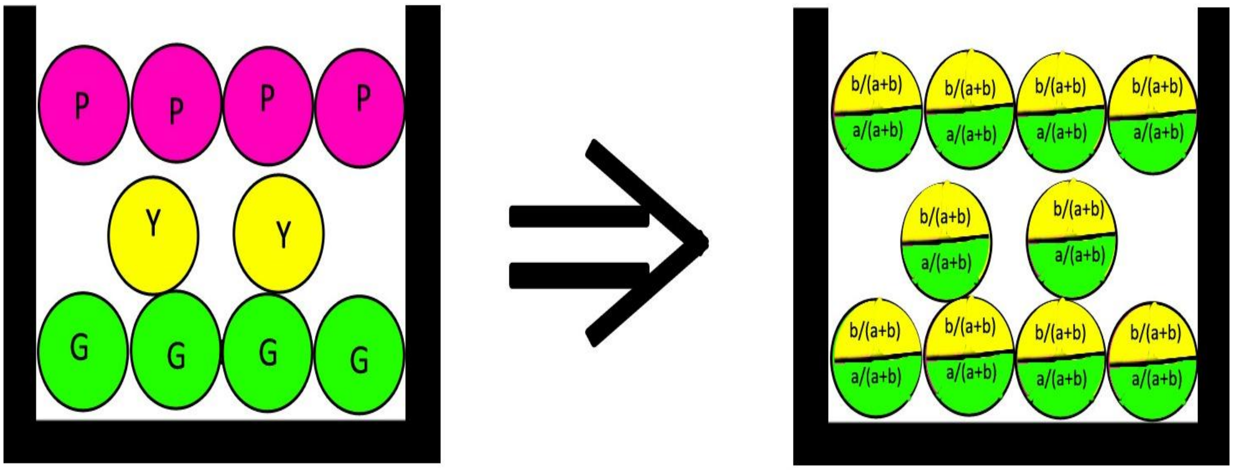
3. CALCULATION AND DISCUSSION

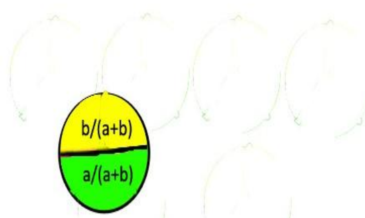
➔ In a bag there are ‘a’ Green balls, ‘b’ yellow balls and ‘c’ pink balls. Balls are drawn one by one and their colour is noted down.
Find the probability that Green balls appear before yellow balls?
Solution: From attribute delocalization, all attributes (Green, yellow, pink) will be delocalized among all the (a + b + c) balls.

So each ball contains $\frac{a}{a+b+c}$ green attributes $\frac{b}{a+b+c}$ yellow attributes and $\frac{c}{a+b+c}$ pink attributes.


P
P
P
P
Y
Y
G
G
G
G
c/(a+b+c)
b/(a+b+c)
a/(a+b+c)

For probability of finding green balls before yellow balls, we draw a ball and see its green portion first and then yellow portion, while ignoring pink portion.





Hence, required probability = $\frac{a}{a+b}$.

Few examples are taken from [3]

Example 1: An urn containing a white and b black balls, k (<a,b) balls are drawn and laid aside, their colour un noted .then one more ball is drawn .find the probability that is it is white.

Solution: Let E_i denote the event that out of the first k balls drawn, i balls are white .let A denote the event that the (k+1)th ball drawn is also white.

We have

$P(E_i)\frac{(a\ C\ i)(a\ C\ k-i)}{(a+b)\ C\ k}(0 \le i \le k)$ and $p(A/E_i) = \frac{a-i}{a+b-k}\ (0 \le i \le k)$

We know that

$$P(A) \sum_{n=1}^{\infty} p(Ej) * p(A/E_j) = f(x) = \sum_{j=1}^{k} \left(\frac{(a\ C\ j)(b\ C\ k-j)}{(a+b)\ C\ k}\right) * \frac{a-j}{a+b-k}$$

$$= \sum_{j=1}^{k} \left(\frac{(a-1\ C\ j)(b\ C\ k-j)}{(a+b) * a+b-1\ C\ k}\right)$$

ISSN 2229-5518

$= \frac{a}{a+b} * \frac{1}{\text{a+b−1 C k}} \sum_{k=0}^{k}((\text{a} - 1\text{ C j})(\text{ b C k} - \text{j}))$

Also $(1+x)^{a-1}(1+x)^{b}$=

$({}^{a-1}C_0 + {}^{a-1}C_1x + {}^{a-1}C_1x + {}^{a-1}C_1x + {}^{a-1}C_1x + \ldots .. + {}^{a-1}C_1x)$

$*({}^{b}C_0 + {}^{b}C_1x + {}^{b}C_2x^2 + \ldots .. + {}^{b}C_bx^b) - (1)$

So $\sum_{k=0}^{k}((\text{a} - 1\text{ C j})(\text{ b C k} - \text{j}))$

=coefficient of x^k on r.h.s of equation (1)

= coefficient of x^k in $(1+x)^{a+b-1} = {}^{a+b-1}C_k$.

Hence p(A) = $\frac{a}{a+b}$.

Alternate solution by attribute delocalization observation:

The white and black attributes are delocalized within all the (a + b) balls.

As fractional count ratio of white ball is $\frac{a}{a+b}$ and black ball is $\frac{b}{a+b}$.

Hence, each ball contains $\frac{a}{a+b}$ white attributes and $\frac{b}{a+b}$ black attribute.

As balls are drawn and laid aside without noticing their colours.

Hence, black attribute and white attribute remains same within all the balls.

Hence, if one ball is drawn in r^{th} attempt then also p(W) =$\frac{a}{a+b}$ and p (B) = $\frac{b}{a+b}$ if and only if for the all previous $(r-1)^{th}$ result is not known.

Example2.A bag contains a white and b black balls. Of these m(≤a) are picked at random and put in an empty bag. From the second bag n(≤m)balls are picked at random and put in a third bag. Find the probability that the third bag contains j(0≤j≤n)white balls.

Solution: Let E_i denote the event that i white (m-i) black balls are transferred from the first to the second bag.

Then $p(E_i)=({}^{a}C_i * {}^{b}C_{m-i})/ {}^{a+b}C_m$ if 0≤i≤a.

$p(E_i)= 0$ if i>a and m-i>b

let F_j denote the event that the third bag contains j white balls.

Then $p(F_j \mid E_i)= 0$ for $i<j$ or $n-j>m-i$.

$p(E_j \mid E_i)=({}^{i}C_j * {}^{m-i}C_{n-j}) / {}^{m}C_n$ for i≥j and $n-j \leq m - j$

we know that

$p(F_j)= \sum_{i=0}^{m} p(E_i)p(F_j \mid E_i) = \sum_{i=j}^{m-n+j} p(E_i)p(F_j \mid E_i)$

ISSN 2229-5518

$=\sum_{i=j}^{m-n+j} \left(\frac{(a\,C\,i)(\,b\,C\,m-i)}{(a+b)\,C\,m}\right) * \left(\frac{(i\,C\,j)(\,m-i\,C\,n-j)}{m\,C\,n}\right)$

Now $\left(\frac{(a\,C\,i)(\,b\,C\,m-i)}{(a+b)\,C\,m}\right) * \left(\frac{(i\,C\,j)(\,m-i\,C\,n-j)}{m\,C\,n}\right) = \frac{a!b!m!(a+b-m)!i!(m-i)!n!(m-n)!}{i!(a-i)!(m-i)!(b-m+i)!(a+b)!(i-j)!j!(n-j)!(m-n-i+j)!m!}$

$=\frac{a!b!}{(a+b)!} * \frac{(a+b-m)!}{(a-i)!(b-m+i)!} * \frac{n!}{j!(n-j)!} * \frac{(m-n)!}{(i-j)!(m-n-i+j)!}$

$=({}^{a+b-m}C_{a-i})*({}^{n}C_{j})*({}^{m-n}C_{i-j}) / ({}^{a+b}C_{a})$

$P(F_j)=\frac{n\,C\,j}{a+b\,C\,a} * \sum_{i=j}^{m-n+j} ({}^{a+b-m}C_{a-i})({}^{m-n}C_{i-j})$

Also $(1+x)^{a+b-n}=(1+x)^{a+b-m}(1+x)^{m-n}$

$=\{ ({}^{a+b-m}C_0)+({}^{a+b-m}C_1x)+({}^{a+b-m}C_2x^2)+ \;.\;.\;.\;.\;.+ {}^{a+b-m}C_{a+b-m}x^m) \}$

$*\{ ({}^{m-n}C_0+ {}^{m-n}C_1x \;.\;.\;.\;.+ {}^{m-n}C_{m-n}x^{m-n}) \}$ (1)

Now

$\sum_{i=j}^{m-n+j} ({}^{a+b-m}C_{a-i})({}^{m-n}C_{i-j})$=coefficient of x^{a-j} on the RHS of(1).

Coefficient of x^{a-j} on the LHS of (1) = ${}^{a+b-n}C_{a-j}$

Therefore

$P(F_j)=(({}^{n}C_j)*({}^{a+b-n}C_{a-j}))/({}^{a+b}C_a) = \frac{n!}{(n-j)!j!} * \frac{(a+b-n)!}{(a-j)!(b-n+j)!} * \frac{a!b!}{(a+n)!}$

$=\frac{(n\,C\,j)(b\,C\,n-j)}{(a+b)\,C\,n}$

Alternate method by attribute delocalization observation:

As we have seen above, putting balls in different urns without noticing any ball is similar to partitioning the urn into portions.

Obviously from **Attribute delocalization observation**, all white attribute and black attribute will be delocalized. Hence each ball inside each partition contains $\frac{a}{a+b}$ white balls and $\frac{b}{a+b}$ black balls.

Picking j white balls from final partition imply picking (n-j) black balls from the partition.

All partitions are done blindly. Hence among final partition any of the balls (among 'a' white balls and 'b' black balls) can be put in the final partition.

That means any n balls among (a + b) balls can be put inside final partition. And in each partition, ball has same configuration that is each ball contains $\frac{a}{a+b}$ white balls and $\frac{b}{a+b}$ black balls.

Hence sample space = ${}^{a+b}C_n$.

Inside final partition we draw j white and n-j black balls but any n balls among (a + b) balls can be put inside final partition .All balls are same in nature. So it can be assumed that we draw j white and n-j black balls in the initial urn having 'a' white balls and b black balls.

No. of events = ${}^{a}C_j * {}^{b}C_{n-j}$.

Hence required probability = $\frac{(n\,C\,j)(b\,C\,n-j)}{(a+b)\,C\,n}$

Example 3: Consider an urn that contain r red and g green balls. A ball is drawn at random and its colour noted. Then the ball, together with c>0 balls of the same colour, are returned to the urn. Suppose n such draws are made from the urn .show that the probability of selecting a red ball at any drawn is $\frac{r}{r+g}$.

Solution: Let R_m denotes the event that the m^{th} ball drawn is red. Let E_i denote the event that i red balls are seen up to the $(m-1)^{th}$ draw. Now, the probability of drawing i red balls in the first i successive draw is

$$\left(\frac{r}{r+g}\right)*\left(\frac{r+c}{r+g+c}\right)*\left(\frac{r+2c}{r+g+2c}\right) - - - - \left(\frac{r+(i-1)c}{r+g+(i-1)c}\right)$$

$$P_i=\left(\frac{r}{r+g}\right)*\left(\frac{r+c}{r+g+c}\right)*\left(\frac{r+2c}{r+g+2c}\right) - - - - -*\left(\frac{r+(i-1)c}{r+g+(i-1)c}\right)$$

$$*\left(\frac{g}{r+g+ic}\right) - - - - \left(\frac{r+(m-i-2)c}{r+g+(m-2)c}\right).$$

Note that P_i is also the probability of drawing i red and (m-i-1) green balls in any given order.it follows that the probability of drawing i red balls in (m-1) draws is ${}^{m-1}C_i p_i$

$$=\frac{(m-1)!r(r+c)\ldots[r+(i-1)c]g(g+c)\ldots[g+(m-i-2)c]}{i!(m-1-i)!(r+g)(r+g+c)\ldots[r+g+(i-1)c]\ldots[r+g+(m-2]c]}$$

$$=\frac{\frac{a(a+1)(a+2)\ldots[a+(i-1)]}{i!}*\frac{b(b+1)\ldots[b+(m-i-2)]}{(m-i-1)!}}{\frac{(a+b)(a+b+1)\ldots(a+b+m-2)}{(m-1)!}}$$

$$=\frac{\binom{-a}{i}\binom{-b}{m-i-1}}{\binom{-a-b}{m-1}} \text{ [where } \binom{-r}{i}=(-1)^i * \frac{r(r+1)\ldots(r+i-1)}{i!} \text{]}$$

Now, $P(R_m)=\sum_{i=0}^{m-1} P(E_i)P(R_m|E_i)$

$$=\sum_{i=0}^{m-1} \frac{\binom{-a}{i}\binom{-b}{m-i-1}}{\binom{-a-b}{m-1}} * \frac{r+ic}{r+g+(m-1)c}$$

$$\sum_{i=0}^{m-1} \frac{\binom{-a}{i}\binom{-b}{m-i-1}}{\binom{-a-b}{m-1}} * \frac{a+i}{a+b+(m-1)} = \sum_{i=0}^{m-1} \frac{a\binom{-a-1}{i}\binom{-b}{m-i-1}}{a+b\binom{-a-b-1}{m-1}}$$

[Since $\binom{-a}{i}(a+i) = (-1)^i \frac{a(a+1)(a+2)\ldots(a+i-1)(a+i-1)(a+i)}{i!}$

$=a\left((-1)^i \frac{a(a+1)(a+2)\ldots(a+i-1)(a+i-1)(a+i)}{i!} \right)=a\binom{-a-1}{i}$]

Hence $P(R_m)=\frac{a}{a+b}\sum_{i=0}^{m-1} \frac{\binom{-a}{i}\binom{-b}{m-i-1}}{\binom{-a-b-1}{m-1}}$

Now we know that for $|x|<1$

$(1+x)^{-a}=1-ax+\frac{a(a+1)}{2!}x^2 - \frac{a(a+1)(a+2)}{3!}X^3 + \ldots$

$=1+\binom{-a}{1}x+\binom{-a}{2}x^2+\binom{-a}{3}x^3+\ldots$

Also $(1+x)^{-a-1}(1+x)^{-b} = (1+x)^{-a-b-1}$

Equating the coefficient of x^{m-1} on both sides ,we get

$$\sum_{i=0}^{m-1} \binom{-a-1}{i}\binom{-b}{m-i-1} = \binom{-a-b-1}{m-1}$$

Hence $P(R_m) = \frac{a}{a+b}$.

Alternate solution by attribute delocalization observation:

From attribute delocalization, we know that as long as ball is not seen, each ball is made of red and green portion in the ratio $\frac{r}{r+g}$ and $\frac{g}{r+g}$ respectively.

When man picks a ball then ball is red with probability $\frac{r}{r+g}$ *and* green with probability $\frac{g}{r+g}$.

And he adds c balls of same colour then it can be assumed that he adds $\frac{r}{r+g}$*c red balls and $\frac{g}{r+g}$*c green balls because probability of happening of red ball is $\frac{r}{r+g}$ and green ball is $\frac{g}{r+g}$.

Due to attribute delocalization, the attributes (red and green) of all balls will be delocalized among all the balls (r + g + c).

Initially we have $\frac{r}{r+g}$*(r+ g) red balls and we added $\frac{r}{r+g}$*c red balls.

So, total no's of red balls $=\frac{r}{r+g}$ (r+g+c)

Similarly total no's of green balls$=\frac{g}{r+g}$ (r+g+c)

Hence, $P(R) = \frac{\frac{r}{r+g}(r+g+c)}{\frac{r}{r+g}(r+g+c)+\frac{g}{r+g}(r+g+c)} = \frac{r}{r+g}$

Question: An urn contains m yellow balls and n green balls. the balls are drawn one at a time until only those of the same colour are left.

Show that the probability that they are all yellow is $\frac{m}{m+n}$.

Solution: Let E_i denote the event that i yellow balls remain in the urn when the last green ball is drawn .This means that out of the first (m+n-i-1) draws, there must be exactly (n-1) green balls and (m-i) yellow balls.

Also $(m + n - i)^{th}$ draw results in a green ball.

Therefore, $P(E_i) = \frac{(n\,C\,n-1)(m\,C\,m-i)}{(m+n)\,C\,(m+n-i-1)} \cdot \frac{1}{i+1} = \frac{n!m!(m+n-i-1)!(i+1)!}{(n-1)!i!(m-i)!(m+n)!(i+1)} = \frac{(m+n-i-1)\,C\,n-1}{(m+n)\,C\,m}$

Let E denote the event that only yellow balls remain in the urn .now $E = E_1 \cup E_2 \cup E_3 \, . \, . \, . \, . \, . \, E_a$,and the E_i's are mutually exclusive.

Therefore ,

$P(E)=P(E_1)+P(E_2)+P(E_3)+\;.\;.\;.\;.+P(E_a)$

$=\frac{1}{m+n\,C\,m}\left({}^{n-1}C_{n-1}+{}^{n}C_{n-1}+\;.\;.\;.+{}^{m+n-2}C_{n-1}\right)$

But ${}^{n-1}C_{n-1}+{}^{n}C_{n-1}+\;.\;.\;.+{}^{m+n-2}C_{n-1}$

$={}^{n}C_{n}+{}^{n}C_{n-1}+\;.\;.\;.+{}^{m+n-2}C_{n-1}$

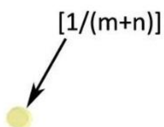
$={}^{n+1}C_{n+1}+{}^{n+1}C_{n-1}+\;.\;.\;.+{}^{m+n-2}C_{n-1}$

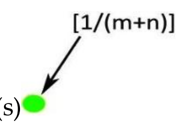
$={}^{n+2}C_{n}+\;.\;.\;.\;.+{}^{m+n-2}C_{n-1}=\;.\;.\;.\;.={}^{m+n-1}C_{n}$

$p(E)=\frac{(m+n-1)!}{n!(m-1)!}*\frac{m!n!}{(m+n)!}=\frac{m}{m+n}$

Alternate solution by attribute delocalization observation:

We know from 2nd explanation of attribute delocalization observation mentioned above (p-11)

Here number(s) of yellow 

trench(s) or green trench(s) 

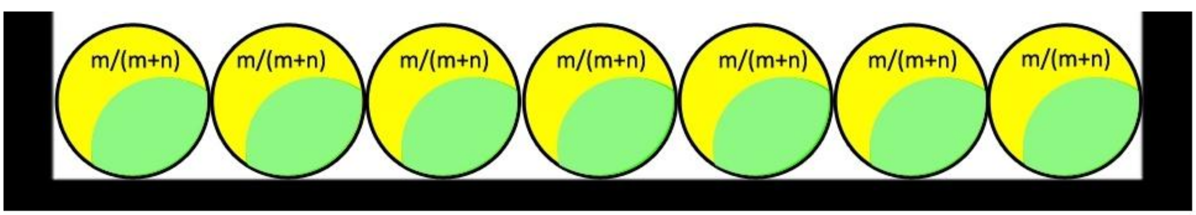
indicates that that many corresponding balls (yellow or green) have been drawn.

Here two groups will be formed.

In group1 all the green balls have been drawn and k yellow ball(s) have been drawn. Where $0\leq k\leq(m-1)$.

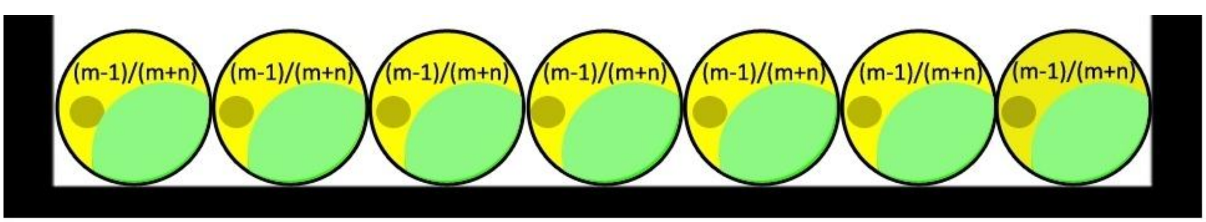
So in group1, we have m cases.

No yellow ball is drawn and all the green balls have been drawn.



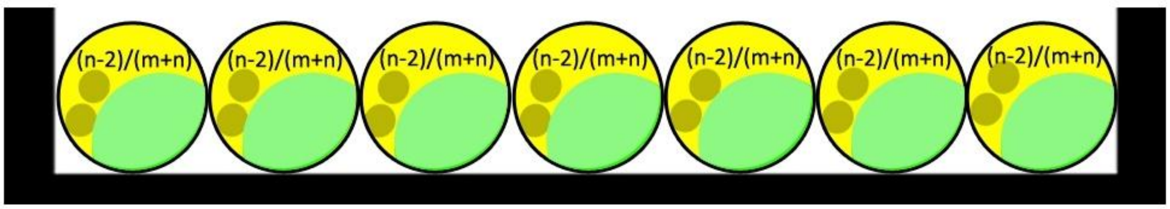
Initial

1 yellow ball is drawn and all the green balls have been drawn.



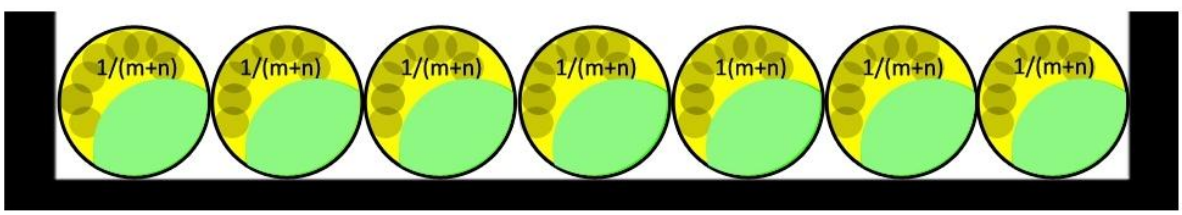
logically contains (m+n) balls

2 yellow balls are drawn and all the green balls have been drawn.



logically contains (m+n) balls

(m-1)yellow balls have been drawn , only one yellow ball remains and all the green balls have been drawn.

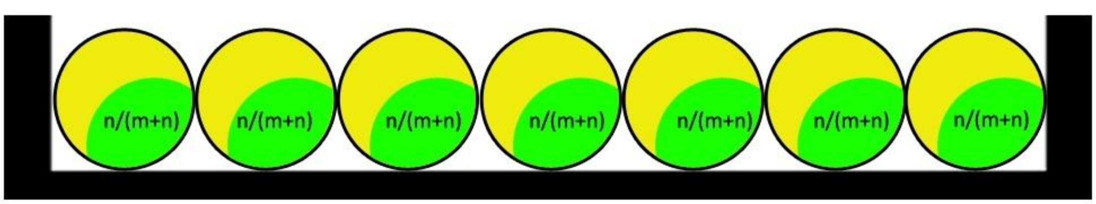


logically contains (m+n) balls

In group2 all the yellow balls have been drawn and k green ball(s) have been drawn. Where $0 \leq k \leq (n-1)$.

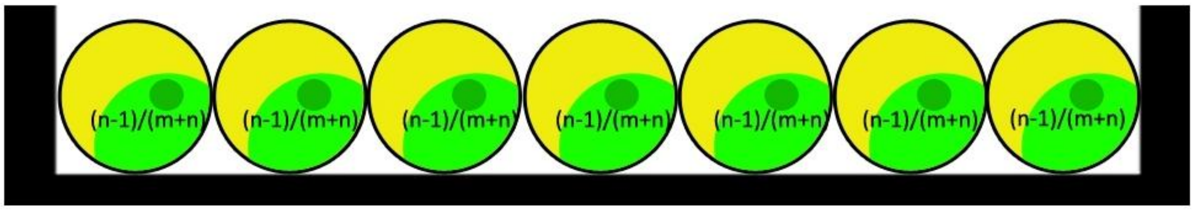
So in group2, we have n cases.

No green ball is drawn and all the yellow balls have been drawn.



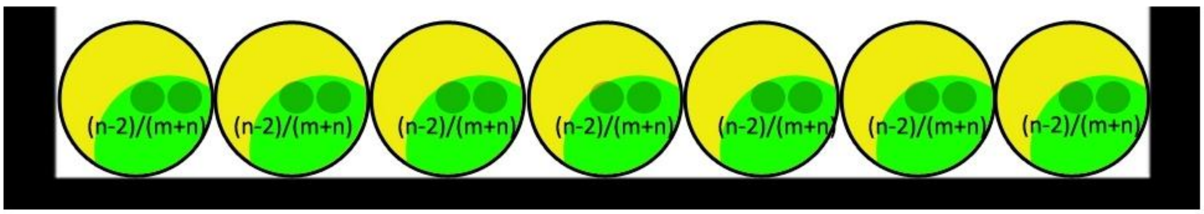
Initial

One green ball is drawn and all the yellow balls have been drawn.



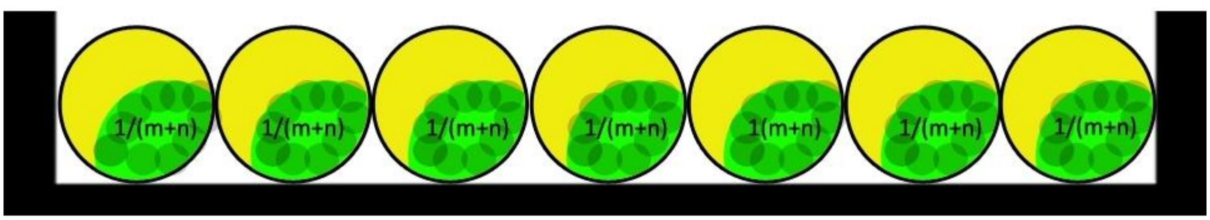
logically contains (m+n) balls

Two green balls are drawn and all the yellow balls have been drawn.



logically contains (m+n) balls

(n-1) green balls have been drawn, only one green ball remains and all the yellow balls have been drawn.



logically contains (m+n) balls

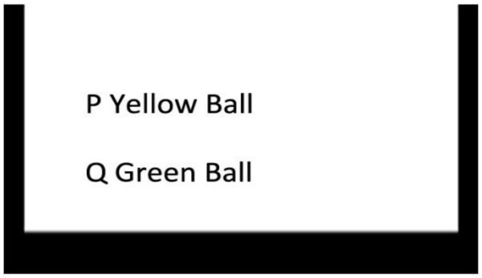
Hence, probability of occurrence of case1 is $\frac{m}{m+n}$.

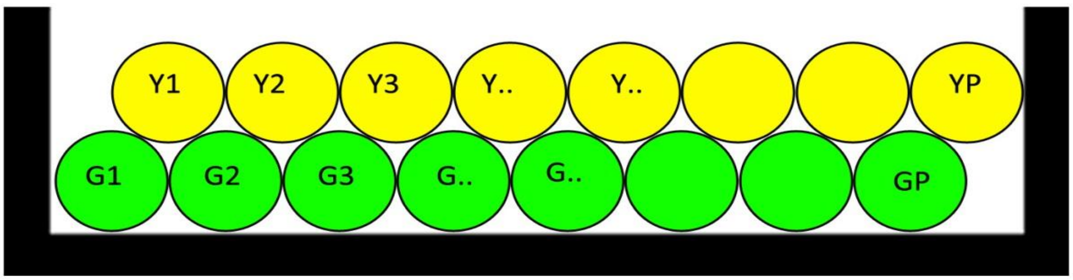
4. Limitation of attribute delocalization observation:

Probability depends upon the concentration not upon the combination.

In a bag there are p yellow balls and q green balls.

Case 1: when balls are distinct.



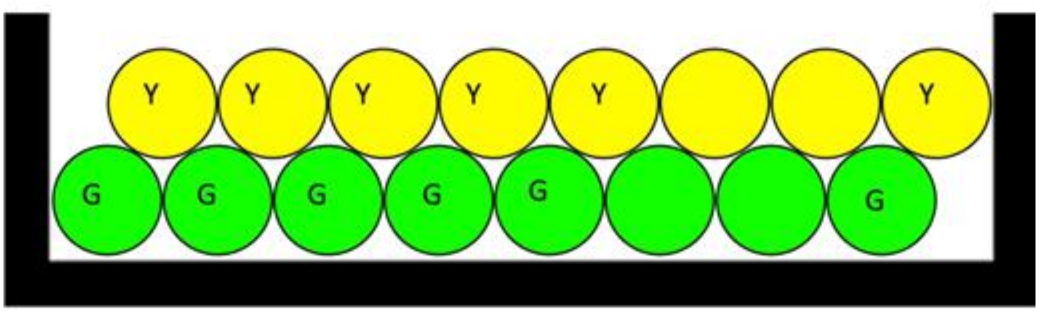


Nos. of selection of yellow balls = $\binom{p}{1}$

Nos. of selection of a ball =$\binom{p+q}{1}$

Hence probability of drawing yellow ball = $\frac{p}{p+q}$(the same result obtained from attribute delocalization observation) .

Case 2: when balls are identical:



No. of selection of a yellow ball = 1.

Total no. of selection of balls = 2 {either green or yellow}.

Here E={G,Y}.

P(Y) = $\frac{1}{2}$ (which is wrong).

Here G and Y are merely symbols.

No. of green balls ≠ no. of yellow balls

That is, Concentration of G ≠ concentration of Y

$\frac{concentration\ of\ yellow}{concentration\ of\ green} = \frac{p}{q}$.

Probability of yellow ball = $\frac{concentration\ of\ yellow}{concentration\ of\ yellow + concentration\ of\ green}$

[Which is nothing but the fractional count ratio.]

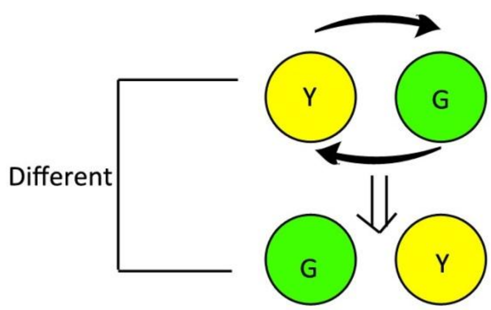
- **If two balls are drawn in one attempt, what is the probability that one is yellow and other is green?**

P(Y,G)=p(G,Y) = $\frac{m}{m+n} * \frac{n}{(m-1)+n}$ (this is wrong)

P(Y,G) denotes probability of drawing yellow in first draw and green in second draw.

p(G,Y) denotes probability of drawing green in first draw and yellow in second draw.

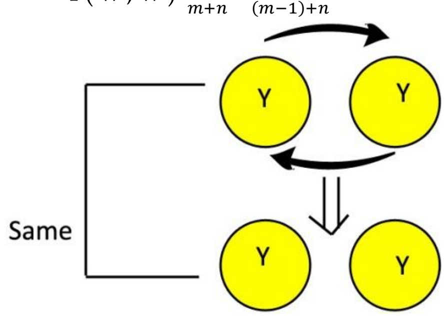
Note: Here alternation of ball matters so we have to multiply by 2! . Draw of yellow ball in first attempt and green ball in second attempt is different from draw of green ball in first attempt and yellow ball in second attempt.



Hence p(Y,G)=p(G,Y)= $\frac{m}{m+n} * \frac{n}{(m-1)+n} * 2!$

- **If two balls are drawn in one attempt, what is the probability that both should be yellow?**
  Obviously,

P(w , w)= $\frac{m}{m+n} * \frac{m-1}{(m-1)+n}$



Here alternation of ball does not yield new set so no need to multiply by 2!.

➔ In above two problems, if balls are identical then no. of ways of selection of two balls = {(G, G), (Y, Y), (G, Y)}
No. of ways of selection of two balls and both are yellow (or one yellow and one green) = 1.
So p(Y, Y) =p (G, Y) =p(Y, G) = $\frac{1}{3}$ (which is wrong because we haven't taken into account concentration. Here concentration of (G,G),(Y,Y) and (G,Y) is not one)

5. Challenging problems:

Few problems are given below, it will be difficult reach to the solution by traditional approach .But we can solve it by attribute delocalization observation easily.

Question1: Bag A contain m white balls and n black balls .

b_i, b_j, ... b_k, balls are drawn from it and put it into the bags B_i, B_j, ... B_k.

from the B_i bag b_{ii}, b_{ij} & b_{ik} balls are drawn and put into the bags B_{ii}, B_{ij} B_{ik}.

From the B_j bag b_{ji}, b_{jj} b_{jk} balls are drawn and put into the bags B_{ji}, B_{jj} B_{jk}.

. . . .

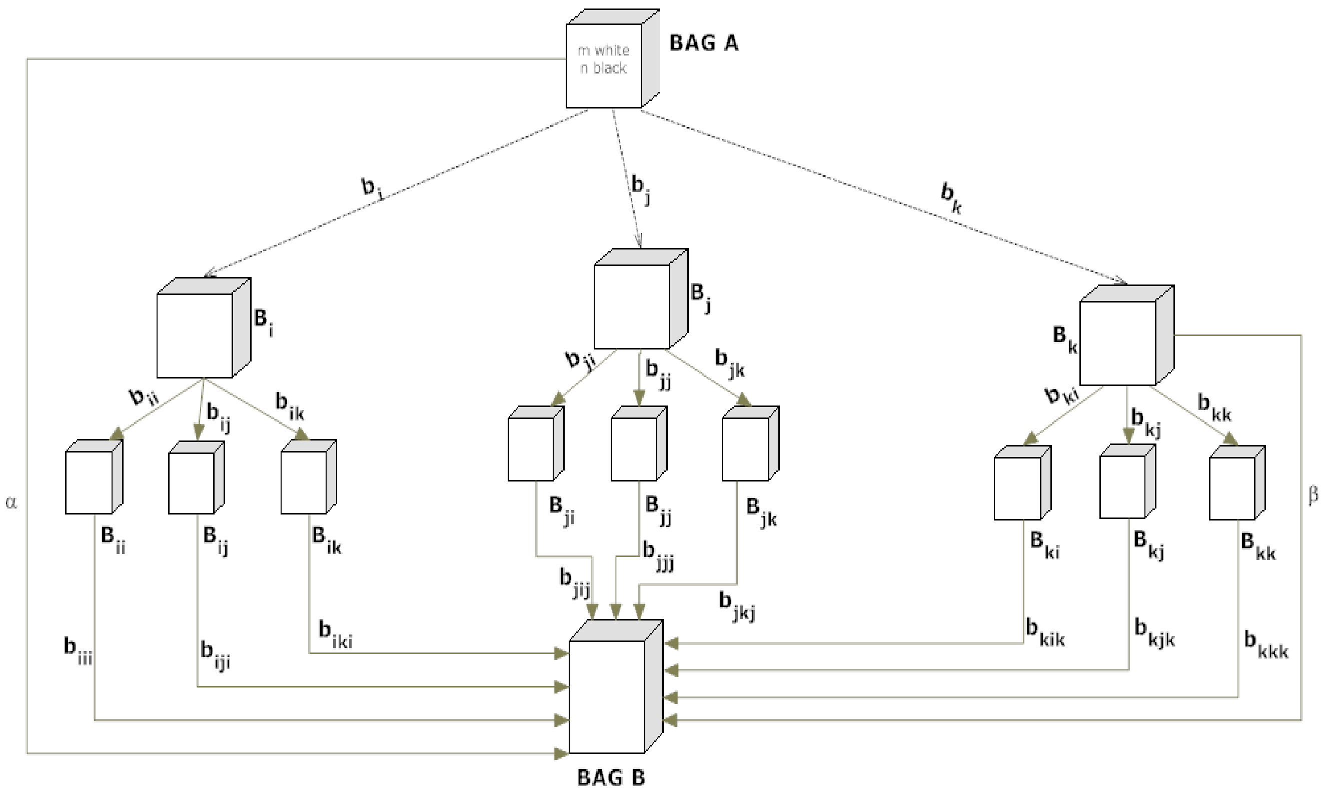
From the B_K bag b_{Ki}, b_{Kj} b_{Kk} balls are drawn and put into the bags B_{Ki}, B_{Kj} B_{Kk}.From the bags B_{ii}, B_{ij} B_{ik}, B_{ji}, B_{jj} B_{jk}.......... B_{Ki}, B_{Kj} B_{Kk} respectively b_{iii} , b_{iji}, b_{iki} , b_{jii}, b_{jij} b_{jki}, ... b_{kik}, b_{kjk}, b_{kkk} balls are drawn and put into the final bag B.

Now Form the initial bag A and from the bag B_k α balls and β balls are drawn respectively and put into the final bag B.

a)what is probability of drawing a white balls from final bag B.

now a person drawn a ball from final bag B and see it's colour and put k additional balls of same colour[all the above experiment has been done without noticing colour].

b) Now, what is probability of drawing white balls from final bag B.?



Solution of a): initially, Bag A contains white balls and black balls in the ratio of m: n and they move randomly through a no of bags and finally resides in Bag B.

Through attribute delocalization observation, it is obvious that in all states (initial, intermediate and final) white and black attribute within any group remains constant.[**It can be visualized as suppose a company produces "mixture" of sugar and salt in the ratio m: n. A wholesaler buys a big bag of this mixture .He puts the mixture into smaller bags for retailers. Now a customer buys this mixture each from company, wholesaler and retailer .In each of the bags he finds that the ratio sugar to salt is m: n. Now if he mixes the mixture of all the bags in that case also the sugar to salt ratio will be m:n.**

Hence probability of any particle (smallest unit) to be sugar =$\frac{m}{m+n}$

and to be salt =$\frac{n}{m+n}$]

Now in bag B .it contains (b_{iii} , b_{iji}, b_{iki} , b_{jii}, b_{jij} b_{jki}, ... b_{kik}, b_{kjk}, b_{kkk})+α+β = μ balls(Let).
Due to attribute delocalization observation if a person draws a ball then it contains $\frac{m}{m+n}$ white attributes and $\frac{n}{m+n}$ black attributes.

Hence probability of drawing a white ball from any bag =$\frac{m}{m+n}$.

b) As each ball is made of white and black portion in the ratio $\frac{m}{m+n}$ and $\frac{n}{m+n}$ respectively.

And he adds k balls of same colour then it can be assumed that he adds k*$\frac{m}{m+n}$ white balls and k*$\frac{n}{m+n}$ black balls.

Due to attribute delocalization all balls attribute (white and black) will be delocalized among all the balls.

Now bag contain k*($\frac{m}{m+n}$) + μ ($\frac{m}{m+n}$) = $\frac{m}{m+n}$ (μ + k) white balls

And $k^*(\frac{m}{m+n}) + \mu\,(\frac{n}{m+n}) = \frac{n}{m+n}(\mu + k)$ black balls

$$p(w) = \frac{(\frac{m}{m+n})(\mu + k)}{(\mu + k)} = \frac{m}{m+n}$$

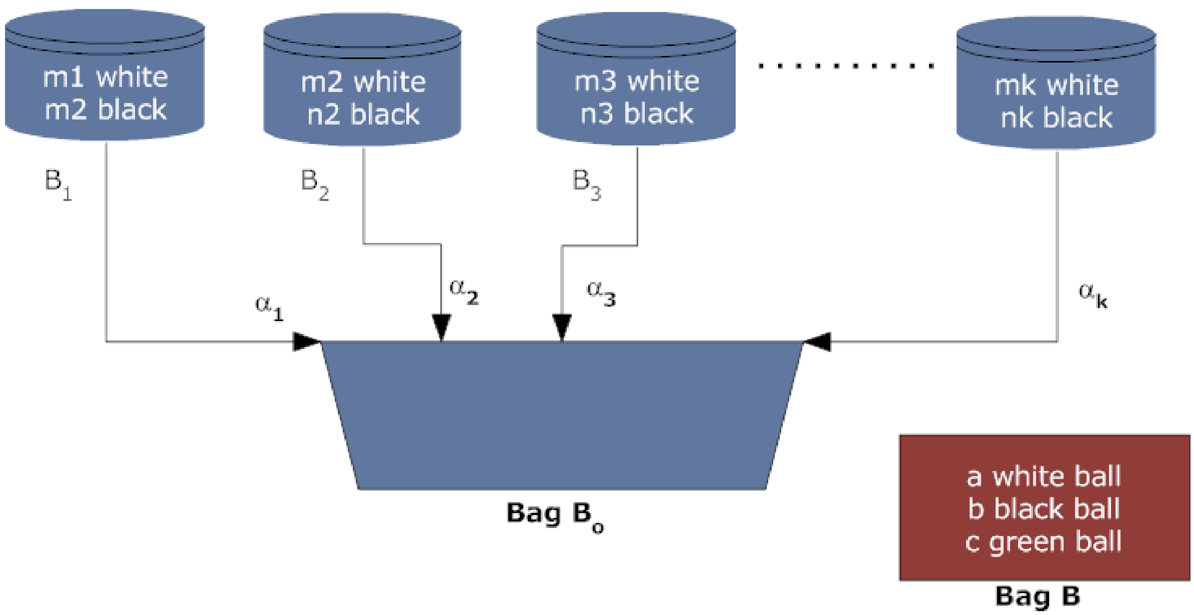
Question 2: Bag B_r contains m_r white balls and n_r black balls .where $r=\{1,2,3,4,\ldots k\}$

$\alpha_1, \alpha_2, \alpha_3 \ldots\ldots \alpha_k$ balls are drawn from the bags $B_1, B_2, B_3 \ldots\ldots B_k$ and put it into bag B_o.

a)find the probability of drawing a white balls from B_o ?

b)A ball is drawn from the bag B_o and put along with k balls of different colour but must be either black or white in the bag B having 'a' white balls ,b black balls and c green balls.

Find the probability of drawing a white ball from bag B?



Solution: Obviously α_r balls of the bag B_r will have $a_r * \frac{mr}{mr+nr}$ white balls and $a_r * \frac{nr}{mr+nr}$ black balls.

Total no of white balls = $\sum_{r=1}^{n}\left(a_r * \frac{mr}{mr+nr}\right)$

Total no black balls= $\sum_{r=1}^{n}\left(a_r * \frac{nr}{mr+nr}\right)$

P(w)= $x(let) = \frac{\sum_{r=1}^{n}\left(a_r * \frac{mr}{mr+nr}\right)}{\sum_{r=1}^{n}(a_n)}$

P(B)= $y(let) = \frac{\sum_{r=1}^{n}\left(a_r * \frac{nr}{mr+nr}\right)}{\sum_{r=1}^{n}(a_r)}$

Here p (w) is white attribute and p(B) is black attribute. Both are present in each ball of bag B. Now a person draws a ball from Bag B, from attribute delocalization it will have p(w) white portion and p(B)black portion.

In the bag B, total no. of white balls = a + p(w) + k*p(B).

Black balls = b + p(B) + k*p(w).

Green balls =c.

P(w)from bag B $=\frac{a + p(w) + k*p(B)}{a+b+c+(k+1)}$

Question3: Urn A contains a red and b black balls and urn B contains c red and d black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random urn B and placed in urn A. Now, if one is drawn from urn A, then find the probability that it is found to be red?

Traditional approach:

Let p(RRR) denote the probability that the balls drawn in first , second and third drawn are red, red and red.

Similarly, p (RBR) denotes the probability that the balls drawn in first second and third draws are red, black and red respectively.

Let E_i denote the probability of drawing a red ball in the i^{th} draw and F_i denote the probability of drawing a black ball in the i^{th} draw.

Then required probability

=P (E1 E2 E3) + P (E1 F2 E3) + P (F1 E2 E3) + P (F1 F2 E3)

OR

P(RRR)+P(RBR)+P(BRR)+P(BBR)

$$\frac{a}{a+b}*\frac{(c+1)}{(c+d+1)}*\frac{a}{a+b} + \frac{a}{a+b}*\frac{d}{(c+d+1)}*\frac{(a-1)}{a+b} + \frac{b}{a+b}*\frac{c}{(c+d+1)}*\frac{a+1}{a+b} + \frac{b}{a+b}*\frac{d+1}{(c+d+1)}*\frac{a}{a+b}$$

Alternate solution by attribute delocalization observation, Transfer of one ball from bag A to bag B can be assumed that $\frac{a}{a+b}$ red balls and $\frac{b}{a+b}$green balls are transferred from urn A to urn B.

Now total red balls in bag B$=\frac{a}{a+b}$ + c.

Within each ball in bag B $\frac{(\frac{a}{a+b})+c}{c+d+1}$ portion will be red.

So transfer of one ball from bag B to bag A can be assumed that $\frac{(\frac{a}{a+b})+c}{c+d+1}$ red balls are transferred .After transfer, bag A has

$\left(\frac{(\frac{a}{a+b})+c}{c+d+1}\right) + \frac{a}{a+b} * (a + b - 1)$ Red balls among total ((a+b-1) + 1) balls.

All red attribute will be delocalized within all the balls.

Within each ball in bag A $\frac{\left(\frac{(\frac{a}{a+b})+c}{c+d+1}\right)+\frac{a}{a+b}*(a+b-1)}{(a+b-1)+1}$ red attribute resides and hence is probability of drawing red ball.

From attribute delocalization each ball in urn A has

$\frac{\left(\frac{(\frac{a}{a+b})+c}{c+d+1}\right)+\frac{a}{a+b}*(a+b-1)}{a+b-1+1}$ red ball.

P(R) = $\frac{\left(\frac{(\frac{a}{a+b})+c}{c+d+1}\right)+\frac{a}{a+b}*(a+b-1)}{a+b}$

Question4:

Urn A contains a red balls, b green balls and c blue balls.

Urn B contains d red balls, e green balls and f blue balls.

Urn C contains g red balls, h green balls and i blue balls.

p nos. of balls are taken out from urn A and put in urn B, similarly q nos. of balls are taken out from urn B and put in urn C.

Now r no. of balls are taken out from urn C and put in urn B.

s no. of balls are taken out from urn B and put into urn A.

{Where p, q, r, s is not exceeding the total balls taken from the original urn}

Finally one ball is drawn from urn A. what is probability of drawing red ball?

Solution:

$$\frac{\left(\frac{\left(\frac{\left(\frac{\left(\frac{a}{a+b+c}\right)*p\ +d}{d+e+f+p}\right)*q\ \ +g}{q+g+h+i}\right)*r+\frac{d(d+e+f-q)}{d+e+f}}{(d+e+f-q+r)}\right)*s+\frac{a(a+b+c-p)}{a+b+c}}{(a+b+c-p+s)}$$

ISSN 2229-5518

Explanation: from attribute delocalization, transfer of p ball from urn A to urn B can be assumed that $(\frac{a}{a+b+c})$*p red balls are transferred.

All attributes inside urn B are delocalized in all the balls, hence each ball in urn B contains $\frac{(\frac{a}{a+b+c})*p+d}{d+e+f+p}$ red balls. So, transfer of q balls from urn B to urn C can be assumed that $(\frac{(\frac{a}{a+b+c})*p+d}{d+e+f+p})$*q red attribute have been transferred. All attributes of urn C are delocalized in all the balls of urn C. Hence all the balls of urn C contain $(\frac{\left(\frac{(\frac{a}{a+b+c})*p+d}{d+e+f+p}\right)*q+g}{q+(g+h+i)})$ red balls. Transfer of r balls from urn C to urn B can be assumed that $(\frac{\left(\frac{(\frac{a}{a+b+c})*p+d}{d+e+f+p}\right)*q+g}{q+(g+h+i)})$*r red balls are transferred.

Before the transfer of r balls, each ball of Bag B has $\frac{d(d+e+f-g)}{d+e+f}$ red balls. All attributes of urn B are delocalized among all the balls ((d + e + f) – q + r) of bag B. So each ball of bag B contains

$$\frac{\left(\frac{\left(\frac{(\frac{a}{a+b+c})*p+d}{d+e+f+p}\right)*q+g}{q+(g+h+i)}\right)*\mathrm{r}+\mathrm{d}\frac{(d+e+f-q)}{d+e+f}}{(d+e+f)-q+r}$$

.transfer of s balls can be assumed

$(\frac{\left(\frac{\left(\frac{(\frac{a}{a+b+c})*p+d}{d+e+f+p}\right)*q+g}{q+(g+h+i)}\right)*\mathrm{r}+\mathrm{d}\frac{(d+e+f-q)}{d+e+f}}{(d+e+f)-q+r})$ *s red balls are transferred.

Before the transfer of S ball bag A has $\frac{a(a+b+c-p)}{a+b+c}$ red balls.

After transfer of S balls from urn B to urn A, all attributes will be delocalized among all the balls (a + b + c - p + s), so each balls contains

$$\frac{\left(\frac{\left(\frac{\left(\frac{(\frac{a}{a+b+c})*p+d}{d+e+f+p}\right)*q+g}{q+g+h+i}\right)*r+\frac{d(d+e+f-q)}{d+e+f}}{(d+e+f-q+r)}\right)*s+\frac{a(a+b+c-p)}{a+b+c}}{(a+b+c-p+s)}$$

Red balls. Hence, the probability of drawing the red balls.

If in this problem ,we increase the no of urns >10 and the value of {a,b,c,d,e,f,g,h,i,p,q,r }>1000 then we can't reach to the solution by traditional approach .

But we can solve it by above observation easily.

6. Conclusion

On the basis of Bernoulli urn scenario, some portion of discrete probability has been studied in this paper. And it was observed that an event and its complements are treated as attributes of sample points. All attributes are delocalized and reside within each sample point in their fractional count ratio. The attribute delocalization can have many possible applications in the field of logical science, computer science and engineering as well as day to day life. To consider event and the complement of event in each sample point is the basic thought of understanding probability.

Due to this observation more complicated problem can be solved easily with certain limitation. This work can be extensively studied for further research in statistics.

7. References

[1] Jaynes Edwin T & Bretthorst G. Larry ,Probability Theory: The Logic of Science, ISBN 0 521 79271 2

[2] http://en.wikipedia.org/wiki/Schr%C3%B6dinger_cat

[3] Course In Mathematics for IIT Joint Entrance Examination, second edition, Tata Mcgraw-Hill Publishing Company Limited New Delhi ISBN 0-07-462221-8.

AUTHOR PROFILE:

Aakash Kumar is Computer Science graduate from KIIT University, Bhubaneswar,Odisha,India.
E-mail : aakashtalk2u@gmail.com
aakashreborn@gmail.com

PH :- +91 90406 11773 , +91 80183 36524

His interests include Algos & DS, Mathematics, Puzzles, and Operating Systems .He is dedicated to do research in field of algorithmic mathematics.